Risk averse water valuation[1]

Andy Philpott
Electric Power Optimization Centre
University of Auckland.
www.epoc.org.nz
(joint work with Michael Ferris and Roger Wets)

[1] EPOC Winter Workshop, September 5, 2013

Motivation: March 2013

WEEK IN REVIEW: Wholesale prices jump, EA launches enquiry, futures soar

Edward White Fri, 08 Mar 2013

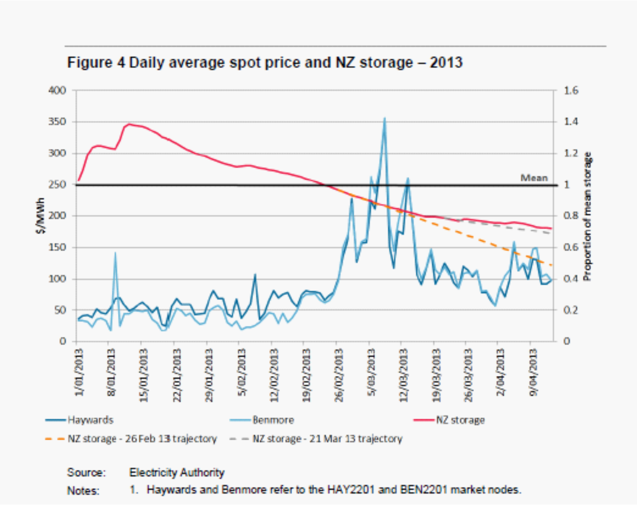
Electricity prices are at an 18-month high after hydro storage continued its steep slide and Contact Energy delayed the restart of its Taranaki Combined Cycle plant by more than two weeks.

The Electricity Authority's market monitoring team this week launched an enquiry into a doubling of wholesale electricity prices since late February.

For the seven days to Sunday, prices at the major Haywards and Benmore nodes averaged $159 a megawatt-hour and $163/MWh respectively – more than twice as much as the week before.

Prices have increased further this week – holding above $200/MWh across Haywards, Benmore and Otahuhu from Tuesday through Thursday, according to NZX Energy. Prices haven't been that high across all three major nodes since August 16, 2011.

Electricity Authority Report, July 29, 2013



High water value partial motivation for single buyer model

Water's 'true price' lost in Labour-Green proposal – academic

Edward White Fri, 26 Apr 2013

New Zealand's water resource would be priced by "fiat" under the Labour-Green proposed electricity market reforms, according to one economist.

The reform of "unfair" pricing of hydro generation is a key aspect of the Labour-Green policy to establish a state-run single-buyer of wholesale electricity.

Labour, in its 13-page policy document released last Thursday, says the three state-owned generators, TrustPower and Contact Energy make "super profits" from the nation's "free water resource."

Victoria University economics professor Lew Evans disputes the assertion that water can be considered a 'free' resource to generators.

"It is not the true price," Evans says.

He says the value of water, and subsequent pricing of electricity, takes into account alternative uses for water, like irrigation, and fluctuations in water supply.

What is the "true price" of water?

The true price is not zero, as it reflects an opportunity cost.

Since water can be stored for later use, agents in the market look at the current price and compare it with predicted prices which might be higher.

But future prices come from future offers that reflect future water values. So there is some sort of endogeneity here.

Different agents will have different views of the future and different aversion to risk. So will this lead to different water prices for different agents?

What is the price of water in competitive equilibrium?

Our goal in this talk is to study the price of water in a *perfectly* competitive equilibrium with risk-averse agents. Of course, perfect competition does not exist in practice, but at least we can model it.

Why bother?

- Compare with market outcomes and try to identify possible weaknesses in the market design and regulation.
- Are prices in the actual market significantly above competitive prices?
- What are the efficiency implications?
- Are there improvements that increase efficiency?

Questions

What are risks?
How do we model these?
How do we deal with different perceptions of risk?

Model risks using coherent risk measures.
Can be different measures for different agents.

Artzner et al, 1999)

Any coherent risk measure of a random cost Z can be expressed as

$$\rho(Z) = \max_{\mu \in \mathcal{D}} \mathbb{E}_{\mu}[Z]$$

where $\mathcal{D}$ is a convex subset of probability measures called the *risk set*. We might have different agents using different risk sets.

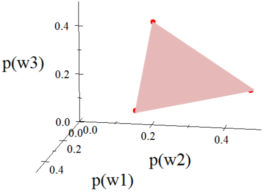
Example: three outcomes

Consider possible cost outcomes

$$Z(\omega_1) < Z(\omega_2) < Z(\omega_3)$$

Let

$$\mathcal{D} = \text{conv}\{(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}), (\frac{1}{4}, \frac{1}{2}, \frac{1}{4}), (\frac{1}{4}, \frac{1}{4}, \frac{1}{2})\}$$



$$\rho(Z) = \max_{\mu \in \mathcal{D}} \mathbb{E}_\mu[Z] = \frac{1}{4} Z(\omega_1) + \frac{1}{4} Z(\omega_2) + \frac{1}{2} Z(\omega_3)$$

Competitive equilibrium under risk

Suppose each agent i has a (possibly different) risk set $\mathcal{D}_i$. Given a random process of electricity prices known now and uncertain in the future each agent minimizes their risk adjusted disbenefit at these prices using their risk set. If the resulting supply and demand plans at these prices clear the market then we say we have a competitive equilibrium.

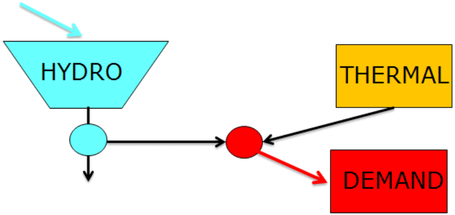
Computing equilibria using EMP
(Ferris and Dirkse)

The EMP framework is now part of GAMS. One feature is modeling Multiple Optimization Problems with Equilibrium Constraints (MOPEC). Each agents a in $\mathcal{A}$ determines her decisions $x_{\mathcal{A}} = (x_a,\ a \in \mathcal{A})$ by solving, independently, an optimization problem,

$$x_a \in \mathrm{argmax}_{x \in \mathbb{R}^{n_a}} f_a(p, x, x_{-a}), \qquad a \in \mathcal{A}, \tag{1}$$

where x_{-a} are actions of competitors, and $p \in \mathbb{R}^d$ are prices that satisfy a global equilibrium constraint that represents market clearing.

Example: three agents, two periods, 10 inflow scenarios



$$
\begin{aligned}
C(v) &= v^2 \\
U(u) &= 1.5u - 0.015u^2 \\
V(x) &= 10\log(0.1x + 0.01) \\
D(d) &= 40d - 2d^2 \\
D_i &= \text{conv}\{(0.55, 0.05, \ldots, 0.05), (0.05, 0.55, \ldots, 0.05), \ldots, (0.05, 0.05, \ldots, 0.55)\}
\end{aligned}
$$

Example: competitive risk neutral equilibrium

t	ω_m	price	storage	release	thermal	profit (T)	profit (H)	welfare (C)	welfare (total)
0		1.336	7.590	6.410	0.668				
1	1	2.539	2.865	5.725	1.269	2.057	20.417	362.283	384.758
1	2	2.053	3.590	6.000	1.027	1.500	19.418	366.863	387.781
1	3	1.696	4.387	6.203	0.848	1.165	18.809	370.264	390.238
1	4	1.431	5.236	6.355	0.716	0.958	18.514	372.809	392.281
1	5	1.231	6.121	6.470	0.616	0.825	18.445	374.746	394.016
1	6	1.076	7.031	6.559	0.538	0.735	18.529	376.252	395.516
1	7	0.953	7.961	6.629	0.477	0.673	18.716	377.446	396.835
1	8	0.855	8.904	6.686	0.427	0.629	18.969	378.411	398.008
1	9	0.774	9.857	6.733	0.387	0.596	19.264	379.204	399.064
1	10	0.706	10.818	6.772	0.353	0.571	19.585	379.866	400.022

Table: Competitive equilibrium with initial storage of 10.

Competitive risk averse equilibrium

t	ω_m	price	storage	release	thermal	profit (T)	profit (H)	welfare (C)	welfare (total)
0		0.816	12.291	6.709	0.408				
1	1	1.118	6.757	6.534	0.559	0.479	14.131	380.898	395.508
1	2	0.987	7.681	6.610	0.494	0.410	14.291	382.175	396.876
1	3	0.882	8.621	6.671	0.441	0.361	14.527	383.202	398.090
1	4	0.796	9.571	6.720	0.398	0.325	14.811	384.042	399.178
1	5	0.725	10.530	6.761	0.363	0.298	15.127	384.740	400.164
1	6	0.665	11.495	6.796	0.333	0.277	15.460	385.328	401.065
1	7	0.614	12.466	6.825	0.307	0.261	15.803	385.829	401.893
1	8	0.571	13.440	6.851	0.285	0.248	16.150	386.262	402.659
1	9	0.532	14.418	6.873	0.266	0.237	16.497	386.638	403.372
1	10	0.499	15.399	6.892	0.249	0.229	16.842	386.968	404.039

Table: Risk averse competitive equilibrium with initial storage of 15

Competitive risk averse social plan

t	ω_m	price	storage	release	thermal	profit (T)	profit (H)	welfare (C)	cost (total)
0		0.816	12.291	6.709	0.408				
1	1	1.118	6.757	6.534	0.559	0.479	14.131	380.898	-395.508
1	2	0.987	7.681	6.610	0.494	0.410	14.291	382.175	-396.876
1	3	0.882	8.621	6.671	0.441	0.361	14.527	383.202	-398.090
1	4	0.796	9.571	6.720	0.398	0.325	14.811	384.042	-399.178
1	5	0.725	10.530	6.761	0.363	0.298	15.127	384.740	-400.164
1	6	0.665	11.495	6.796	0.333	0.277	15.460	385.328	-401.065
1	7	0.614	12.466	6.825	0.307	0.261	15.803	385.829	-401.893
1	8	0.571	13.440	6.851	0.285	0.248	16.150	386.262	-402.659
1	9	0.532	14.418	6.873	0.266	0.237	16.497	386.638	-403.372
1	10	0.499	15.399	6.892	0.249	0.229	16.842	386.968	-404.039

Table: Risk averse social planning solution with initial storage of 15

Competitive risk averse equilibrium

t	ω_m	price	storage	release	thermal	profit (T)	profit (H)	welfare (C)	welfare (total)
0		1.317	7.580	6.420	0.658				
1	1	2.545	2.858	5.722	1.272	2.053	20.280	362.407	384.740
1	2	2.057	3.582	5.998	1.029	1.492	19.277	367.002	387.771
1	3	1.700	4.378	6.202	0.850	1.156	18.664	370.413	390.233
1	4	1.434	5.226	6.353	0.717	0.948	18.366	372.965	392.279
1	5	1.233	6.111	6.469	0.616	0.814	18.295	374.908	394.017
1	6	1.077	7.022	6.558	0.539	0.724	18.378	376.418	395.520
1	7	0.955	7.951	6.629	0.477	0.661	18.564	377.615	396.840
1	8	0.856	8.894	6.686	0.428	0.617	18.816	378.582	398.015
1	9	0.775	9.847	6.733	0.387	0.584	19.111	379.377	399.071
1	10	0.707	10.808	6.772	0.353	0.559	19.432	380.040	400.031

Table: Risk averse competitive equilibrium with initial storage of 10

Competitive risk averse social plan

t	ω_m	price	storage	release	thermal	profit (T)	profit (H)	welfare (C)	cost (total)
0		1.545	7.710	6.290	0.773				
1	1	2.472	2.948	5.763	1.236	2.125	21.918	360.888	-384.931
1	2	2.004	3.682	6.028	1.002	1.601	20.968	365.307	-387.876
1	3	1.660	4.486	6.224	0.830	1.286	20.401	368.589	-390.276
1	4	1.404	5.340	6.370	0.702	1.090	20.138	371.050	-392.277
1	5	1.210	6.229	6.482	0.605	0.963	20.090	372.927	-393.980
1	6	1.060	7.142	6.568	0.530	0.878	20.189	374.390	-395.457
1	7	0.940	8.073	6.637	0.470	0.818	20.385	375.553	-396.756
1	8	0.844	9.018	6.692	0.422	0.775	20.644	376.495	-397.914
1	9	0.765	9.972	6.738	0.382	0.743	20.944	377.270	-398.957
1	10	0.699	10.934	6.776	0.349	0.719	21.267	377.919	-399.905

Table: Risk averse social planning solution with initial storage of 10.

Contracts to decrease risk
(Heath and Ku 2004, Ralph and Smeers, 2013)

Suppose we introduce 10 contracts to trade risk, one for each scenario.

We can model a contract to trade risk as an Arrow-Debreu security.

Contract ω_m has a payoff at time 1 of \$1 if scenario ω_m occurs. Agent i buys $W_i(\omega_m)$ (possibly -ve) of these contracts at time 0 at prices $\mu(\omega_m)$, and so pays $\sum_m \mu(\omega_m) W_i(\omega_m)$.

The market for contracts must clear, so

$$\sum_i W_i(\omega_m) = 0.$$

Competitive risk-averse equilibrium

t	ω_m	price	storage	release	thermal	profit (T)	profit (H)	welfare (C)	welfare (total)
0		1.545	7.710	6.290	0.773				
1	1	2.472	2.948	5.763	1.236	-1.232	18.320	367.842	384.931
1	2	2.004	3.682	6.028	1.002	-0.039	19.568	368.347	387.876
1	3	1.660	4.486	6.224	0.830	0.700	20.309	369.267	390.276
1	4	1.404	5.340	6.370	0.702	1.405	21.045	369.826	392.277
1	5	1.210	6.229	6.482	0.605	1.999	21.663	370.319	393.980
1	6	1.060	7.142	6.568	0.530	2.510	22.189	370.758	395.457
1	7	0.940	8.073	6.637	0.470	2.956	22.647	371.153	396.756
1	8	0.844	9.018	6.692	0.422	3.353	23.050	371.511	397.914
1	9	0.765	9.972	6.738	0.382	3.708	23.410	371.838	398.957
1	10	0.699	10.934	6.776	0.349	4.031	23.735	372.139	399.905

Table: Risk-averse competitive equilibrium with risk trading.

Trading risk

t	ω_m	price	trade (T)	trade (H)	trade (C)
0					
1	1	0.280	1.658	0.768	-2.426
1	2	0.080	3.375	2.966	-6.341
1	3	0.080	4.429	4.274	-8.703
1	4	0.080	5.330	5.274	-10.604
1	5	0.080	6.051	5.938	-11.989
1	6	0.080	6.647	6.366	-13.013
1	7	0.080	7.153	6.627	-13.781
1	8	0.080	7.593	6.772	-14.364
1	9	0.080	7.980	6.832	-14.813
1	10	0.000	8.327	6.834	-15.161

Table: Risk trading between three agents in equilibrium

Competitive risk-averse social plan

t	ω_m	price	storage	release	thermal	profit (T)	profit (H)	welfare (C)	cost (total)
0		1.545	7.710	6.290	0.773				
1	1	2.472	2.948	5.763	1.236	2.125	21.918	360.888	-384.931
1	2	2.004	3.682	6.028	1.002	1.601	20.968	365.307	-387.876
1	3	1.660	4.486	6.224	0.830	1.286	20.401	368.589	-390.276
1	4	1.404	5.340	6.370	0.702	1.090	20.138	371.050	-392.277
1	5	1.210	6.229	6.482	0.605	0.963	20.090	372.927	-393.980
1	6	1.060	7.142	6.568	0.530	0.878	20.189	374.390	-395.457
1	7	0.940	8.073	6.637	0.470	0.818	20.385	375.553	-396.756
1	8	0.844	9.018	6.692	0.422	0.775	20.644	376.495	-397.914
1	9	0.765	9.972	6.738	0.382	0.743	20.944	377.270	-398.957
1	10	0.699	10.934	6.776	0.349	0.719	21.267	377.919	-399.905

Table: Risk-averse social planning solution with using intersection of risk sets.

Social plan equals competitive equilibrium

Proposition

Suppose the market for risk trading is complete, and $\mathcal{D}_0 = (\cap_{i\in\mathcal{H}}\mathcal{D}_i) \cap (\cap_{j\in\mathcal{T}}\mathcal{D}_j) \cap (\cap_{c\in\mathcal{C}}\mathcal{D}_c)$ *is nonempty. Let the optimal solution of social planner be* $(u^*, v^*, x^*, d^*, \mu^*)$*, and let the shadow prices of the market clearing constraints be* π_1^**, and* $\mu_m^*\pi_2^*(\omega_m)$*. Finally let* Z^* *denote the vector of disbenefits of each agent:*

$$
\begin{aligned}
Z_i^*(\omega_m) &= -(\pi_1^* U_i(u_1^*(i)) + \pi_2^*(\omega_m) U_i(u_2^*(i,\omega_m)) - V_i(x_2^*(i,\omega_m))) \\
Z_j^*(\omega_m) &= -\pi_1^* v_1^*(i) - \pi_2^*(\omega_m) v_2^*(i,\omega_m) + C_j(v_1^*(j)) + C_j(v_2^*(j,\omega_m)) \\
Z_c^*(\omega_m) &= \pi_1^* d_1^*(c) + \pi_2^*(\omega_m) d_2^*(c,\omega_m) - D_c(d_1^*(c)) - D_c(d_2^*(c,\omega_m)).
\end{aligned}
$$

Then there exists a set of risk contract purchases $W^* = (W_i^*, W_j^*, W_c^*)$ *so that* $(u^*, v^*, x^*, d^*, \mu^*, Z^*, W^*, \pi^*)$ *is a competitive risk-averse equilibrium with risk trading.*

Dynamic risked Walrasian equilibrium

Consider a set of agents $i \in \mathcal{A}$ and stochastic process of demands d and inflows for each $i \in \mathcal{A}$ defined by a scenario tree with nodes $n \in \mathcal{N}$ and leaves $\mathcal{L}$. Suppose that each agent $i \in \mathcal{A}$ is endowed with a dynamic coherent risk measure defined by a state-dependent risk set $\mathcal{D}_i(n)$, $n \in \mathcal{N} \setminus \mathcal{L}$.

A *dynamic risked equilibrium* is a stochastic process of prices $\{\pi(n) \mid n \in \mathcal{N}\}$ adapted to the tree, and for each agent i a stochastic process of actions $\{x_i(n) \mid n \in \mathcal{N}\}$, with the property that

$$0 \leq \sum_{i \in \mathcal{A}} x_i(n) - d(n) \perp \pi(n) \geq 0, \ n \in \mathcal{N}$$

and $x_i(n)$ is a solution to the stochastic optimization problem where agent i at node n minimizes risked future disbenefit at prices $\{\pi_n \mid n \in \mathcal{N}\}$ and risk set $\mathcal{D}_i(n)$.

Dynamic risked Walrasian equilibrium

Consider a set of agents $i \in \mathcal{A}$ and stochastic process of inflows and demands for each $i \in \mathcal{A}$ defined by a scenario tree with nodes $n \in \mathcal{N}$. Suppose that each agent $i \in \mathcal{A}$ is endowed with a dynamic coherent risk measure defined by a state-dependent risk set $\mathcal{D}_i(n)$, $n \in \mathcal{N} \setminus \mathcal{L}$.
If in each node $n \in \mathcal{N}$, there is a complete market for risk, then there is a dynamic system risk measure defined by $\mathcal{D}_0(n)$, $n \in \mathcal{N}$, with the property that minimizing system risked future disbenefit with $\mathcal{D}_0$ gives the same set of actions $\{x_i(n) \mid n \in \mathcal{N}\}$ as a dynamic risked equilibrium.

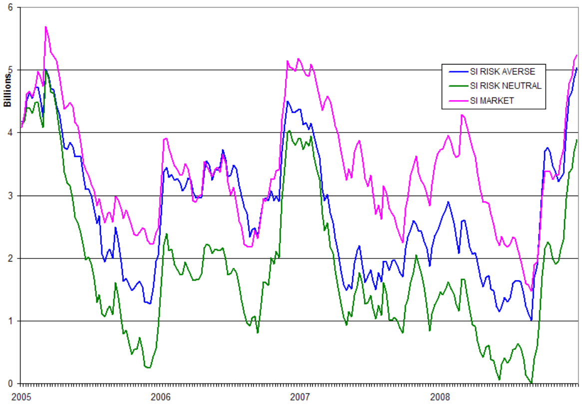
SDDP with coherent risk measures
(P. and de Matos, 2011, 2012, Shapiro 2012)

We sample M inflow outcomes ω_t per stage, and set $\alpha = \frac{1}{M}$. Stage problem is:

$$
\begin{array}{lll}
Q_t(x_{t-1},\omega_t) = \min_{X,U} & c_t^\top V_t + \theta_{t+1} & \\
 & & \\
 & X_t = x_{t-1} - U_t + \omega_t - S_t, & [\pi_t(\omega_t)] \\
 & U_t + V_t \geq d_t, & \\
 & \theta_{t+1} \geq \alpha_i + \beta_i^\top X_t, & i = 1,2,\ldots,k \\
 & & \\
 & U_t \in \mathcal{U}, \quad V_t \in \mathcal{V}, \quad X_t \in \mathcal{X}. &
\end{array}
$$

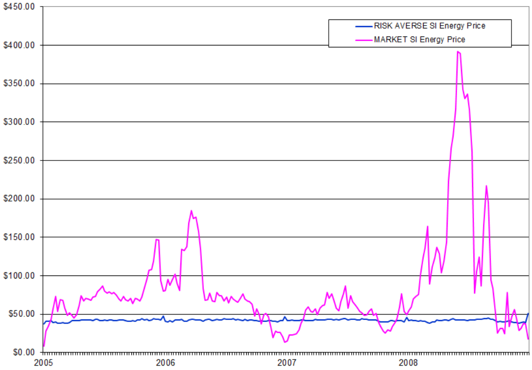
For each ω_t, identify the worst outcome out of M, and compute the cut with probabilities chosen from $\mathcal{D}$ that assign a higher weight to the worst outcome.

Reservoir levels



South Island storage trajectories for risk neutral ($\lambda = 0$) and risk-averse ($\lambda = 0.5$) policies both assuming independent inflows. Market has \$14.29M more storage than ($\lambda = 0.5$) on December 31, 2008.

Risk averse prices



Weekly average South Island prices from risk averse model with $\lambda = 0.5$ (blue) compared with historical Benmore prices (pink).

Results

	Annual thermal fuel cost ($M)		
	MARKET	λ=0	λ=0.5
2005	451.79	369.03	393.94
2006	490.99	442.73	451.07
2007	492.51	438.90	462.45
2008	508.49	439.74	437.32

Annual fuel cost for different levels of risk aversion. The risk neutral solution ($\lambda = 0$) incurs load shedding cost of $41.52M in 2008. The risk-averse solution ($\lambda = 0.5$) incurs no load shedding.

	Annual costs ($M)				
	MARKET	λ=0.5 Fuel	λ=0.5 Shortage	EVPI	Efficiency loss
2005	451.79	393.94	0.00	18.98	38.87
2006	490.99	451.07	0.00	10.93	28.99
2007	492.51	462.45	0.00	15.87	14.18
2008	508.49	437.32	0.00	16.36	40.51

Annual productivity gain for risk averse social plan. We account for residual storage in the 2008 efficiency loss, so total efficiency losses of the market over four years are $122.55 M with respect to this plan ($105.61 M for the risk-neutral plan).

Conclusions and caveats

- There will be other sources of risk besides inflows that contribute to differences in behaviour.
- The appropriate risk measure for a social planner requires knowledge of agent's risk sets, and complete risk markets.
- In practice, markets for risk in hydro inflows will be incomplete. Problems of existence and uniqueness of competitive equilibria.
- Competitive partial equilibrium with incomplete markets is becoming computable, and opening up new avenues of research.
- What is the competitive price of water? If agents are risk averse, then it could be quite different from observed values.