Contracting, risk, and information

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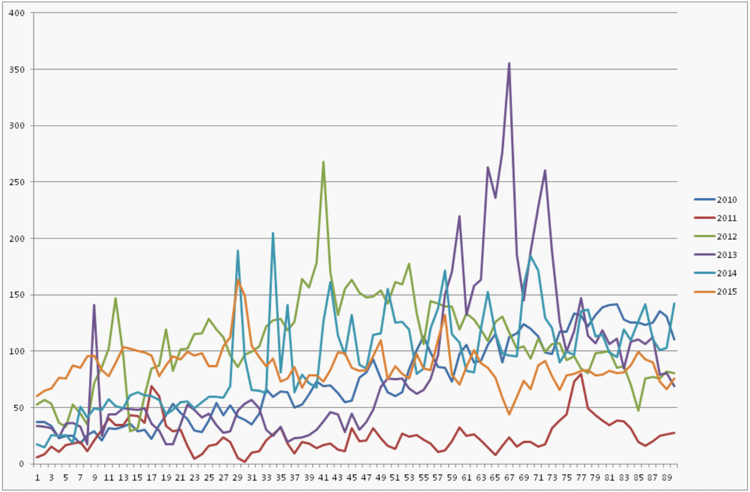
Spot prices in January, February, March



Daily average Benmore prices over the first 90 days of each calendar year.

Electricity hedge contract prices

- Background:

  EA Hedge Market Development Project Consulation, May 2014
  WAG Energy Link Reports, May 2014, August 2014

- Are electricity contract prices traded at a premium to spot?
- This talk gives some details relating to the EPOC submission on this topic.
- We focus on

  The effect of imperfect competition.
  The effect of different information.

Summary

Summary

Previous work

- Allaz and Vila, 1993
- Bessembinder and Lemmon, 2002
- Bushnell, 2007
- Anderson and Hu, 2008

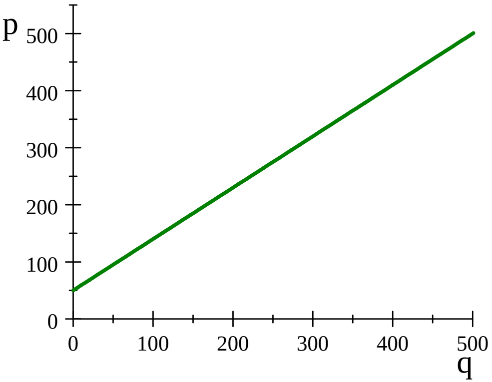
Supply function equilibrium

[Klemperer and Meyer, 1989, Holmberg and Newbery, 2010]

Consider a duopoly of two generators at a single node supplying a single partially hedged load. Suppose generators are identical with capacity K and marginal cost $\$c/\text{MWh}$. Demand is uncertain in the range on $[0, 2K]$ with a small probability of load shedding. Suppose a price cap of VOLL$=V$. The symmetric Nash equilibrium in supply functions is

$$T(q) = c + \frac{V - c}{K} q$$

Example



Example with $V = 500$, $c = 50$, $K = 500$, $Q = 400$

Contracts and offer prices

Now suppose that each generator writes Q contracts for differences at price π. The payoff to a generator when dispatched quantity q at p is

$$pq - cq + Q(\pi - p) = p(q - Q) - cq + Q\pi.$$

Traders offer energy at low prices to ensure being dispatched $q > Q$. This has the right motivation, but there is no obligation to supply Q. Contracts must purchase Q at spot prices, so if generator is supplying at least Q at spot prices then she is covered. But she could be happy with dispatch less than Q if p is low. The key term is $p(q - Q)$, which incentivizes high p when dispatched $q > Q$, and low p when dispatched $q < Q$.

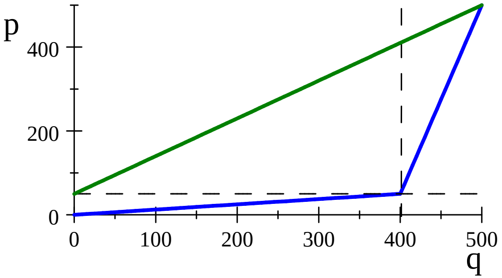
Contracts and supply function equilibrium

A symmetric Nash equilibrium in supply functions is

$$T(q) = \begin{cases} p(q), & q \leq Q \\ c + \frac{(V-c)(q-Q)}{K-Q}, & q > Q \end{cases}$$

where $p(q)$ is any strictly increasing linear function through (Q, c).

Example



Example with $V = 500$, $c = 50$, $K = 500$, $Q = 400$

Observations

- Generators offer below marginal cost up to the contract quantity and above marginal cost above the contract quantity.
- Contracts make the supply function equilibrium more competitive.
- Prices when uncontracted are high. Prices when contracted are low.
- If one looks at contract prices and historical spot prices in the contracted period then one sees a contract premium as the contracts were settled with price expectations before the contract. After contracting spot prices drop. This difference has nothing to do with risk.

What about ex-ante expectations?

Consider a retailer about to negotiate contracts with two identical generators. They settle a contract price π based on expected future prices $\mathbb{E}[P]$ in equilibrium. In the absence of strategic bidding in the spot market, future prices are independent of the contract level. If there are any risk-neutral speculators then we expect contract prices to equal expected spot prices. If not, then a speculator can arbitrage and improve her expected profits. If $\pi > \mathbb{E}[P]$ then she would sell a contract to the consumer at price $(\pi + \mathbb{E}[P])/2 < \pi$, and on average make

$$(\pi + \mathbb{E}[P])/2 - \mathbb{E}[P] = (\pi - \mathbb{E}[P])/2 > 0.$$

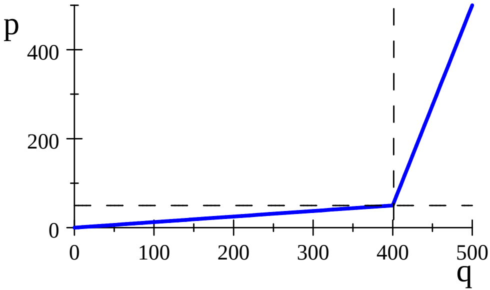
What about strategic generators?

Many authors (following Allaz and Vila) claim that generator market power does not change the result that arbitrage will yield $\pi = \mathbb{E}[P]$, where $\mathbb{E}[P]$ is the expected price at the equilibrium contracted level. The argument is that any difference between π and $\mathbb{E}[P]$ provides an opportunity for arbitrage, and so this will drive these differences to zero. But we see a premium in the NZEM: why do speculators not reduce this to zero?

Example

Consider uniform demand on [0,1000], and contracts of 400 for each generator. Expected spot price is

$$\frac{1}{5}(500+50)\frac{1}{2}+\frac{4}{5}(0+50)\frac{1}{2}=\$75$$



Example with $V=500$, $c=50$, $K=500$, $Q=400$.

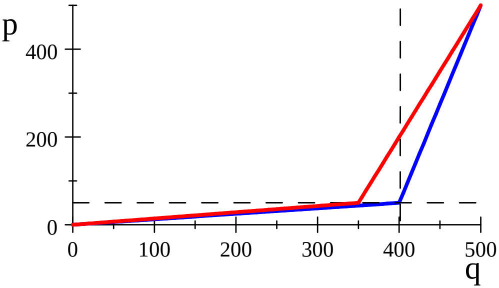
Duopoly model

The retailer's expected cost of meeting demand is

$$
\begin{aligned}
\mathbb{E}[DP] + \text{contract cost} &= \int_0^{400} \frac{cq}{Q} 2q \frac{1}{500} dq \\
&\quad + \int_{400}^{500} \left(c + \frac{(V-c)(q-Q)}{K-Q} \right) 2q \frac{1}{500} dq \\
&\quad + (100-75)400 + (100-75)400 \\
&= 61\,666 + 20000 \\
&= 81666
\end{aligned}
$$

Speculator model

Suppose contract price = \$100 >\$75. Speculator offers retailer 100 MWh of contracts at an undercutting price \$99/MWh. We expect the retailer will prefer these contracts to those of the generators who will now sell only 350MWh of contracts. They will then bid the red curve in the spot market.



Example with $V = 500$, $c = 50$, $K = 500$, Blue $Q = 400$, Red $Q = 350$.

Example

The new expected spot price is

$$\mathbb{E}[P] = 0.7 * \frac{50}{2} + 0.3 * \frac{550}{2} = 100$$

The retailer's expected cost of meeting demand is now

$$\begin{aligned}\mathbb{E}[DP] + \text{contract cost} &= \int_0^{350} \frac{cq}{350} 2q \frac{1}{500} dq \\ &+ \int_{350}^{500} \left(c + \frac{(V-c)(q-350)}{K-350} \right) 2q \frac{1}{500} dq \\ &+ (99-100)100 + (100-100)700 \\ &= 81666 - 100\end{aligned}$$

Payoff to speculator is negative (-100).

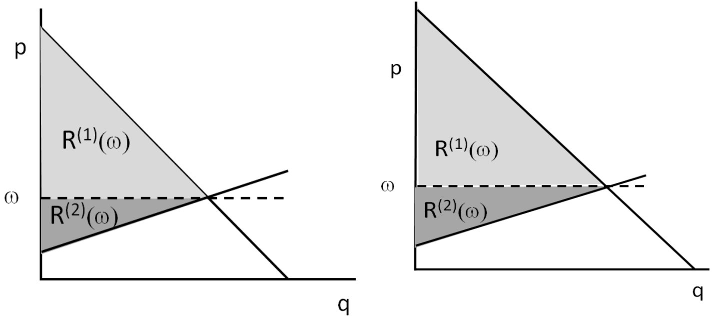
The takeaway message

- Contract levels do not affect the offering behaviour of price-taking generators. The contract payments can be seen as separate transactions to reduce risk.
- With price-taking agents, risk neutral speculators will arbitrage any contract premiums that might exist.
- With price-setting agents, contract levels affect offering behaviour. Risk-neutral speculators then might find it hard to make money.

Summary

Payoffs in the spot market

Consider a perfectly competitive spot market containing a single purchaser (1) and a single generator (2), with identical utility functions $U(x) = x - \frac{1}{2}bx^2$.



In price realization ω the purchaser has payoff $R^{(1)}(\omega)$ and generator makes profit $R^{(2)}(\omega)$.

Risk and contracts

Suppose there are only two price outcomes, high (ω_H) and low (ω_L). Let ρ be the probability of a high price. The agents negotiate a contract Q at price π. The purchaser expected utility is

$$\begin{aligned}\Pi^{(1)} &= \rho U(R^{(1)}(\omega_H) - Q(\pi - \omega_H)) \\ &+(1-\rho)U(R^{(1)}(\omega_L) - Q(\pi - \omega_L))\end{aligned}$$

The generator expected utility is

$$\begin{aligned}\Pi^{(2)} &= \rho U(R^{(2)}(\omega_H) + Q(\pi - \omega_H)) \\ &+(1-\rho)U(R^{(2)}(\omega_L) + Q(\pi - \omega_L))\end{aligned}$$

Given π, each agent chooses $Q(\pi)$ to maximize their expected

utility.

Example 1

Suppose $\rho = \frac{1}{2}$, $\omega_H = 2$, $\omega_L = 1$, $R^{(1)}(\omega_H) = R^{(2)}(\omega_L) = 1$, $R^{(1)}(\omega_L) = R^{(2)}(\omega_H) = 4$. Since

$$U(x) = x - \frac{1}{2}bx^2, \quad U'(x) = 1 - bx$$

$$\Pi^{(1)} = \frac{1}{2}U(1 - Q(\pi - 2)) + \frac{1}{2}U(4 - Q(\pi - 1))$$

$$\Pi^{(2)} = \frac{1}{2}U(4 + Q(\pi - 2)) + \frac{1}{2}U(1 + Q(\pi - 1))$$

Example 1 optimality conditions

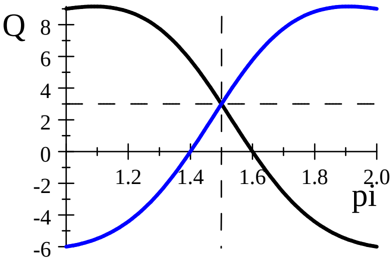
$$\frac{\partial \Pi^{(1)}}{\partial Q} = -\frac{1}{2}(\pi - 2)(1 - b(1 - Q(\pi - 2))) - \frac{1}{2}(\pi - 1)(1 - b(4 - Q(\pi - 1))) = 0$$

$$\frac{\partial \Pi^{(2)}}{\partial Q} = \frac{1}{2}(\pi - 2)(1 - b(4 + Q(\pi - 2))) + \frac{1}{2}(\pi - 1)(1 - b(1 + Q(\pi - 1))) = 0$$

Example 1: Solution bid and offer curves

$$Q^{(1)} = -\frac{(2\pi + 6b - 5\pi b - 3)}{5b + 2\pi^2 b - 6\pi b}$$

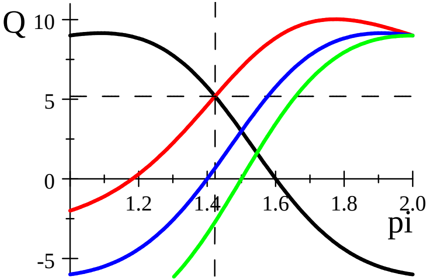
$$Q^{(2)} = \frac{(2\pi + 9b - 5\pi b - 3)}{5b + 2\pi^2 b - 6\pi b}$$



Example contract equilibrium when $b = 0.1$. Equilibrium contract price π= 1.5. solves $Q^{(1)}(\pi) = Q^{(2)}(\pi)$.

Example 2: Different generator payoffs

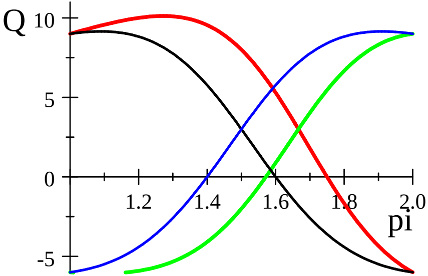
Suppose $\rho = \frac{1}{2}$, $\omega_H = 2$, $\omega_L = 1$, $R^{(1)}(\omega_H) = 1$, $R^{(1)}(\omega_L) = 4$, $R^{(2)}(\omega_L) = 1$. Equilibrium contracts shown for $R^{(2)}(\omega_H) = 8$, $R^{(2)}(\omega_H) = 4$, $R^{(2)}(\omega_H) = 1$.



Example contract equilibria when $R^{(2)}(\omega_H) = 8$ (red), $R^{(2)}(\omega_H) = 4$ (blue), $R^{(2)}(\omega_H) = 1$ (green).

Example 3: Changing probability of high price

Suppose $\rho = \frac{2}{3}$, $\omega_H = 2$, $\omega_L = 1$, $R^{(1)}(\omega_H) = 1$, $R^{(1)}(\omega_L) = 4$, $R^{(2)}(\omega_L) = 1$. Equilibrium contracts shown for $R^{(2)}(\omega_H) = 4$.



Example contract equilibria when $\rho = \frac{1}{2}$ (black and blue) and $\rho = \frac{2}{3}$.(red and green)

Summary

Price speculation

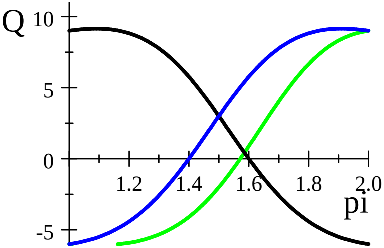
In the previous examples both purchaser and generator had the same ρ. In other words they shared beliefs on future price levels. Consider a model in which these beliefs differ. So purchaser has ρ_1 and generator has ρ_2.

$$\begin{aligned}\Pi^{(1)} &= \rho_1 U(R^{(1)}(\omega_H) - Q(\pi - \omega_H)) \\ &\quad +(1-\rho_1)U(R^{(1)}(\omega_L) - Q(\pi - \omega_L))\end{aligned}$$

$$\begin{aligned}\Pi^{(2)} &= \rho_2 U(R^{(2)}(\omega_H) + Q(\pi - \omega_H)) \\ &\quad +(1-\rho_2)U(R^{(2)}(\omega_L) + Q(\pi - \omega_L))\end{aligned}$$

Example 3: generator varies probability of high price

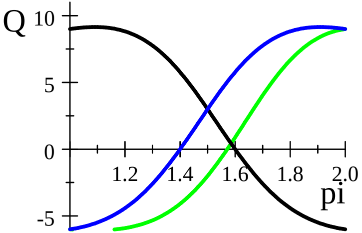
Suppose $\omega_H = 2$, $\omega_L = 1$, $R^{(1)}(\omega_H) = R^{(2)}(\omega_L) = 1$, $R^{(1)}(\omega_L) = R^{(2)}(\omega_H) = 4$. Let $\rho_1 = \frac{1}{2}$ and $\rho_2 = \frac{2}{3}$.



Example contract equilibria when $\rho = \frac{1}{2}$ (black and blue) and $\rho = \frac{2}{3}$.(green)

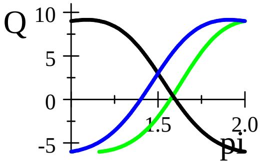
Using Information

If the purchaser conjectures ρ_1 and the generator conjectures ρ_2 then the purchaser should be able to improve his contract bid curve using information from the generator's bid.



The purchaser with known ρ_1 constructs a bid curve (black) by choosing the best point $(\pi(\rho_2), Q(\rho_2))$ to cross each generator offer curve: e.g. the blue curve (if $\rho_2 = \frac{1}{2}$) and the green curve (if $\rho_2 = \frac{2}{3}$).

Supply function equilibrium (limited information)

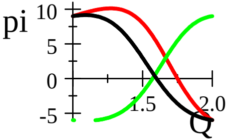


Consider the generator with private information ρ_2. She offers a contract curve (e.g. green or blue for different ρ_2) to the purchaser of the form $\pi(t, \rho_2)$, $Q(t, \rho_2)$. The retailer with belief ρ_1 wants the best outcome on this curve so he chooses t to maximize

$$\begin{aligned}\Pi^{(1)} &= \rho_1 U(R^{(1)}(\omega_H) - Q(t, \rho_2)(\pi(t, \rho_2) - \omega_H)) \\ &\quad +(1-\rho_1) U(R^{(1)}(\omega_L) - Q(t, \rho_2)(\pi(t, \rho_2) - \omega_L))\end{aligned}$$

Setting $\left[\frac{\partial}{\partial t}\Pi^{(1)}\right]_{\rho_1} = 0$ gives maximum payoff.

Supply function equilibrium (limited information)



Consider the purchaser with private information ρ_1. He offers a contract curve (e.g. black or red for different ρ_1) to the purchaser of the form $\pi(\rho_1, t)$, $Q(\rho_1, t)$. The generator wants the best outcome on this curve so she chooses t to maximize

$$\begin{aligned}\Pi^{(2)} &= \rho_2 U(R^{(2)}(\omega_H) - Q(\rho_1, t)(\pi(\rho_1, t) - \omega_H)) \\ &\quad + (1-\rho_2) U(R^{(2)}(\omega_L) - Q(\rho_1, t)(\pi(\rho_1, t) - \omega_L))\end{aligned}$$

Setting $\left[\frac{\partial}{\partial t}\Pi^{(2)}\right]_{\rho_2} = 0$ gives maximum payoff.

Supply function equilibrium

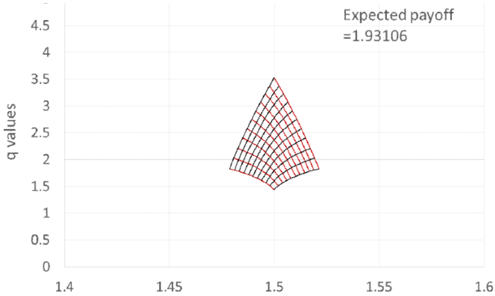
The conditions

$$\left[\frac{\partial}{\partial t}\Pi^{(1)}\right]_{\rho_1} = 0$$

$$\left[\frac{\partial}{\partial t}\Pi^{(2)}\right]_{\rho_2} = 0$$

involve derivatives $\frac{\partial}{\partial t}Q(t,\rho_2)$, $\frac{\partial}{\partial t}\pi(t,\rho_2)$, $\frac{\partial}{\partial t}Q(\rho_1,t)$, $\frac{\partial}{\partial t}\pi(\rho_1,t)$ in nonlinear equations with $Q(\rho_1,\rho_2)$ and $\pi(\rho_1,\rho_2)$. We can solve these simultaneous equations numerically on a grid using GAMS/CONOPT.

Supply function equilibrium results (limited information)



Offer and bid curves for contracts forming a supply function equilibrium.

Supply function equilibrium (full information)

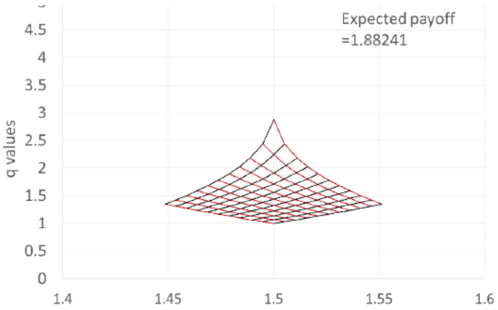
Repeat the above analysis with

$$\rho = \frac{\rho_1 + \rho_1}{2}$$

$$\begin{aligned}\Pi^{(1)} &= \rho U(R^{(1)}(\omega_H) - Q(\rho_1, \rho_2)(\pi(\rho_1, \rho_2) - \omega_H)) \\ &\quad +(1-\rho)U(R^{(1)}(\omega_L) - Q(\rho_1, \rho_2)(\pi(\rho_1, \rho_2) - \omega_L))\end{aligned}$$

$$\begin{aligned}\Pi^{(2)} &= \rho U(R^{(2)}(\omega_H) + Q(\rho_1, \rho_2)(\pi(\rho_1, \rho_2) - \omega_H)) \\ &\quad +(1-\rho)U(R^{(2)}(\omega_L) + Q(\rho_1, \rho_2)(\pi(\rho_1, \rho_2) - \omega_L))\end{aligned}$$

Supply function equilibrium results (full information)



Expected payoff is lower than when information not used. Indeed lower than the original model when each agent just selects his/her own curve irrespectice of others.

What does all this mean?

- In electricity markets where agents offer strategically the determinants of contract prices are more complex than just risk aversion.
- One often hears anecdotes that exercise of market power increases spot prices in February to stimulate contracting at high contract price levels. This talk is not about that effect.
- Contracts affect offering behaviour, and this affects prices and incentives for arbitrageurs.
- Contracting using SFE model can exploit counterparty's private information.
- Preliminary results indicate that it could often be worse in equilibrium to try and use the counterparty's partially revealed information. More to say here.
- The Electricity Authority needs to start getting serious about using these sorts of models to understand the real causes of "risk" premia in contracts.

This is the end

THE END

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