Norus | Electricity modelling in Brazil

Vitor de Matos
CEO

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About Norus

Tech company specialized in the electricity market. By connecting our academic and practical know-hows, we build innovative solutions for our clients improving their operational and decision-making process.

WHO TRUST US?

3 digital products and more than 100 clients in Brazil

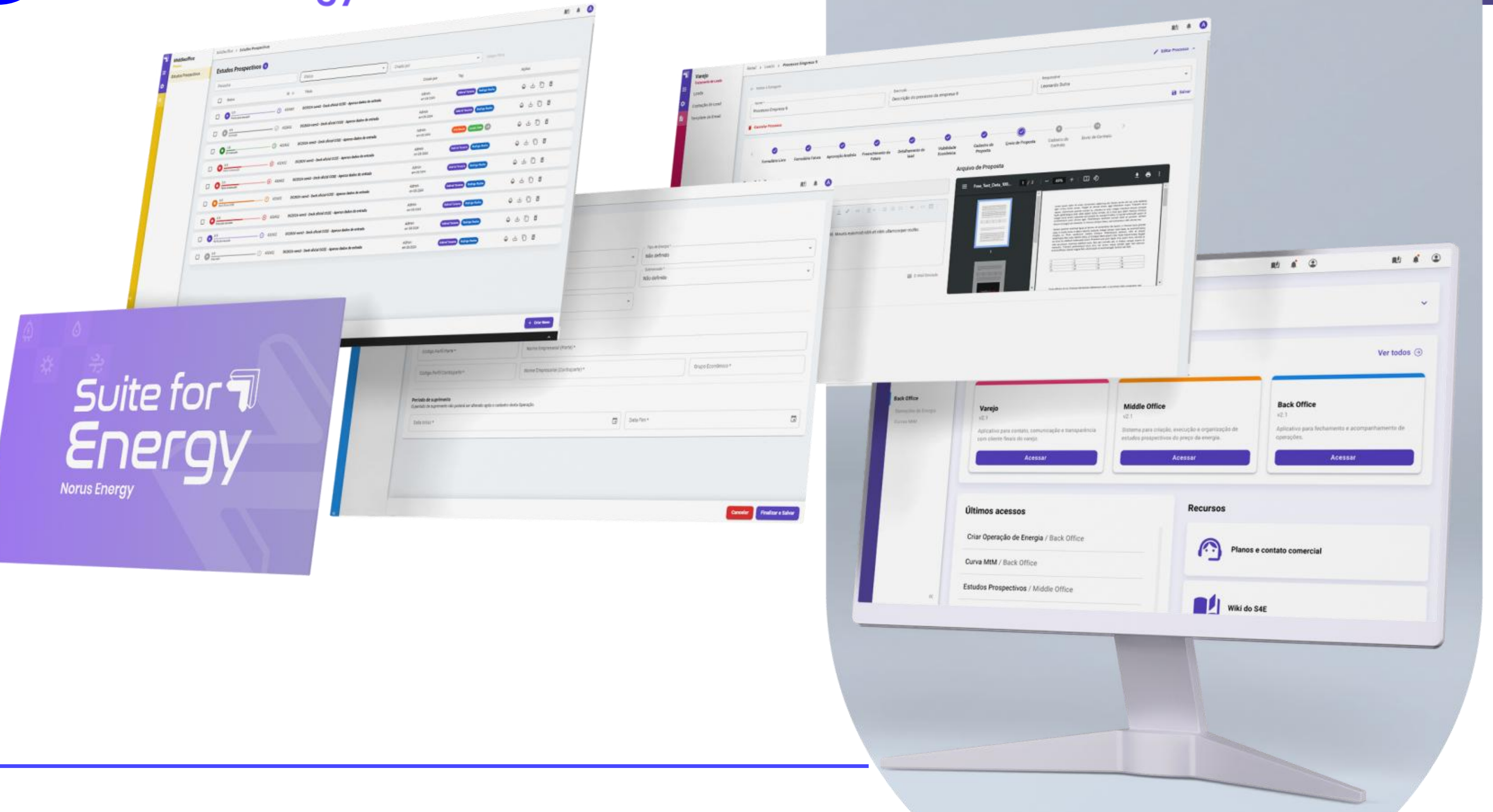
19 out of 20 top market players are our clients

(in total of trading volume in 2024)

+10 R&D and consultancy projects

Suite for Energy

Norus Energy



An ecosystem for apps to the energy market

An environment where all apps developed by Norus and future partner will be delivered.

What are we aiming for?

To connect all trading and strategic decision in energy markets through the whole journey making it easier for different areas to exchange information natively, operating under an integrated framework.

Power Fusion

Norus Energy

Norus' Scheduling Model

Allow specialized studies and analyzing the Impact of new methodologies

Some characteristics

- State-of-the-art SDDP algorithm
- Hydro and Thermal plants are modelled individually
- High flexibility in defining time steps and stages
- Allow different modelling in the forward and backward pass

All development and analysis in this presentation are made using Power Fusion

About the Brazilian Electricity Market

Brazilian Market Design and Trading

Regulated Market

Residential and small Commercial consumers

Larger consumer that didn’t opt to migrate to the free market

Tariffs

Jun/24 to May/25 → 42.2 GWavg = 59.4%

In 2024 more consumers were allowed to opt-in

Expected that by 2028 all consumers will be able to chose

Free Market

Larger consumers

Retail trading companies

Negotiated prices

Jun/24 to May/25 → 28.8 GWavg = 40.6%

Considering that we face highly volatile market, as clients migrate to the Free Market the bigger is the need for long-term contracts increasing also the trading volumes.

Total volume of registerd contracts was 165 TWh in Mar/2025 which is approxametly 3 times de consumption 57 TWh

The Brazilian Case

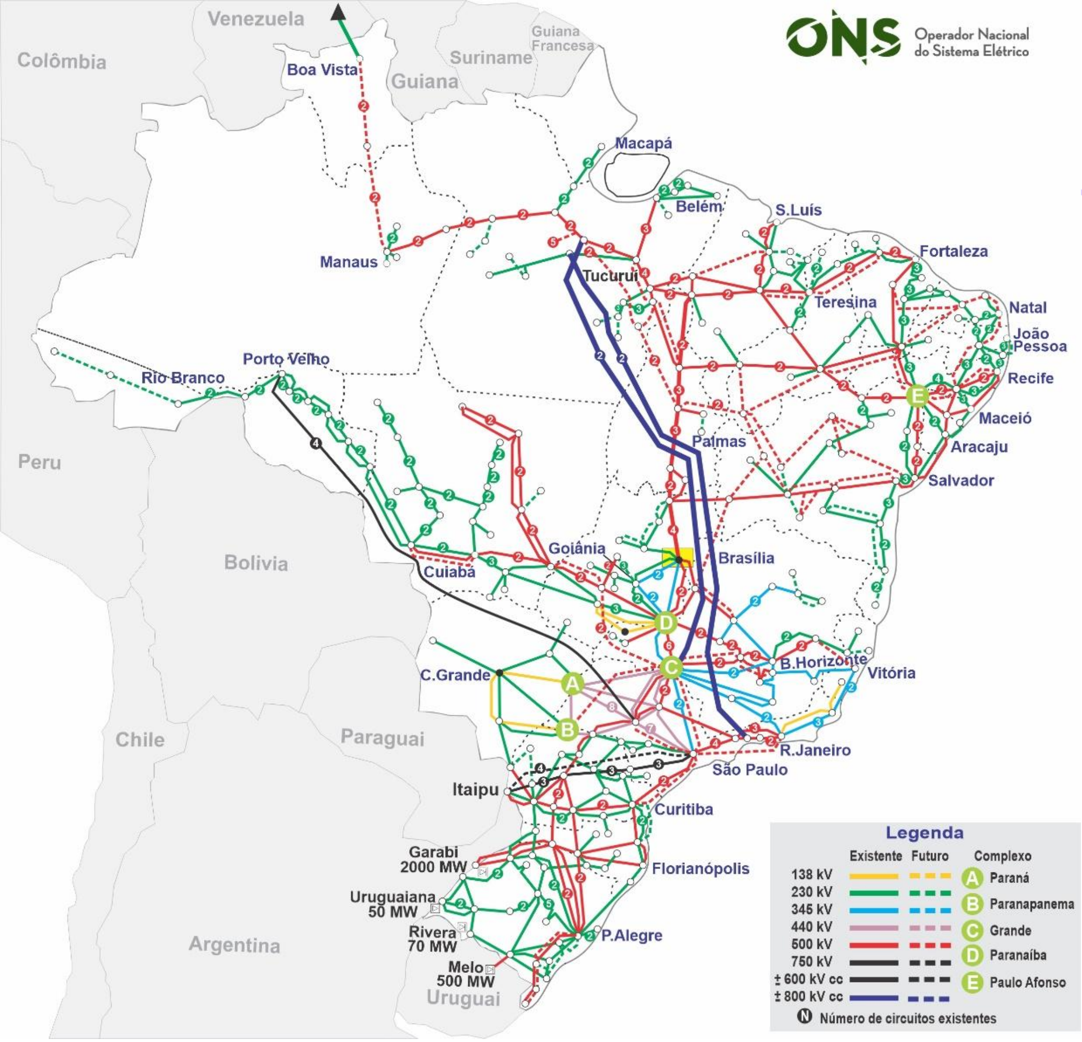
Generation

Most generation is operated centrally by ONS

Some numbers

- Total consumption in 2024 was 594 TWh
- **Total generation capacity is 236 GW**
- Hydro generation: **108,2 GW ~ 46%**
- Wind and Solar: **90,3 GW ~38%**

The Brazilian Case



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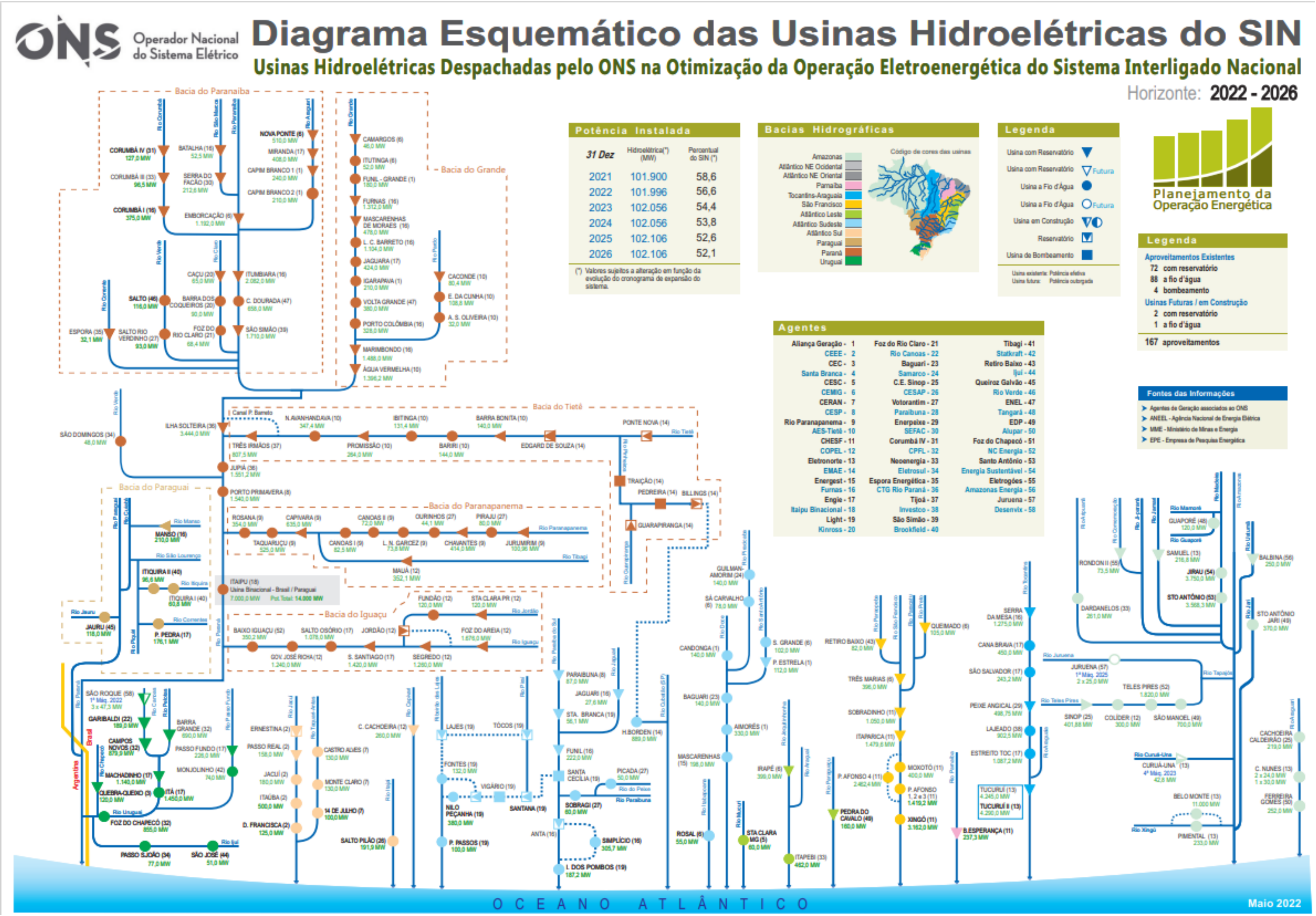

Brazilian Hydrothermal (wind+solar) Scheduling

Main aspects

- Large reservoirs that require longer planning periods
- Uncertainty on Inflows – Other uncertainties that are ignored: demand, wind, solar, …
- Coupled in Time – Reservoirs storage and inflow scenario impact the future decisions
- Coupled in Space – River cascades and Network transmission system conect decisions
- Large problem – Number of variables and constraints

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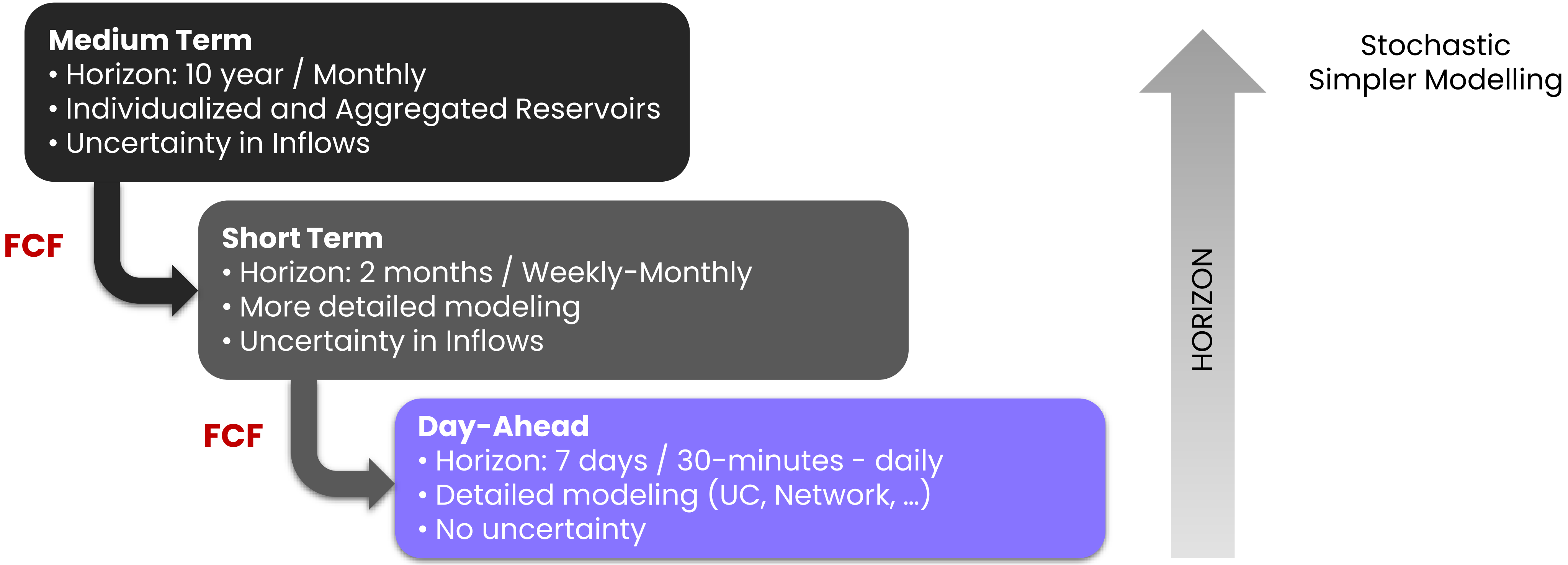
Strategy: Divide to Conquer!

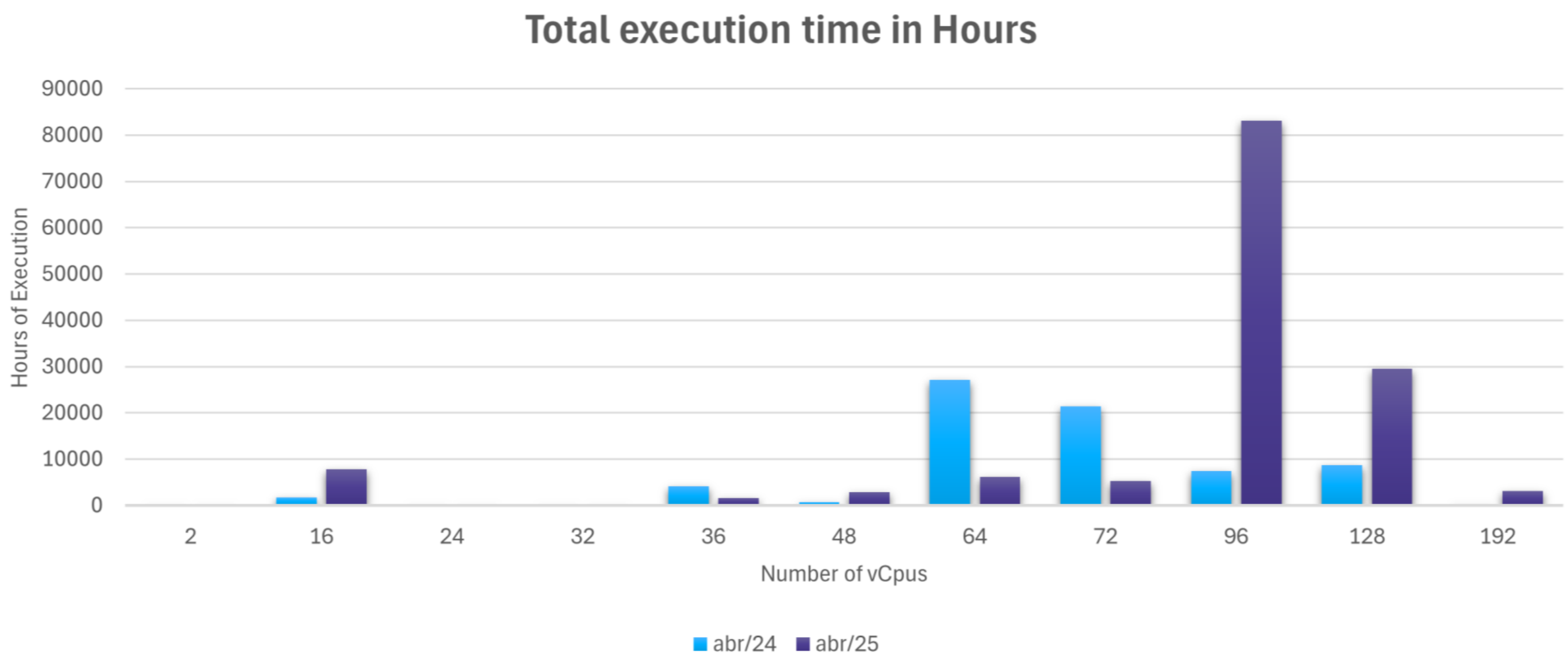
Brazilian Hydrothermal Scheduling

- Scheduling is divided in time with different horizons
- Models are coupled through Future Cost Functions
- Models need to be approved by a commission
- They are used to define operation and pricing in Brazil



Modelling is always changing

Example: In January/2025 the Medium Term started to officially model hydro power plants as individual plants in the first 12 months and then aggregated from stage 13 onward, resulting in a relevant increase in computational burden



Why models are always changing?

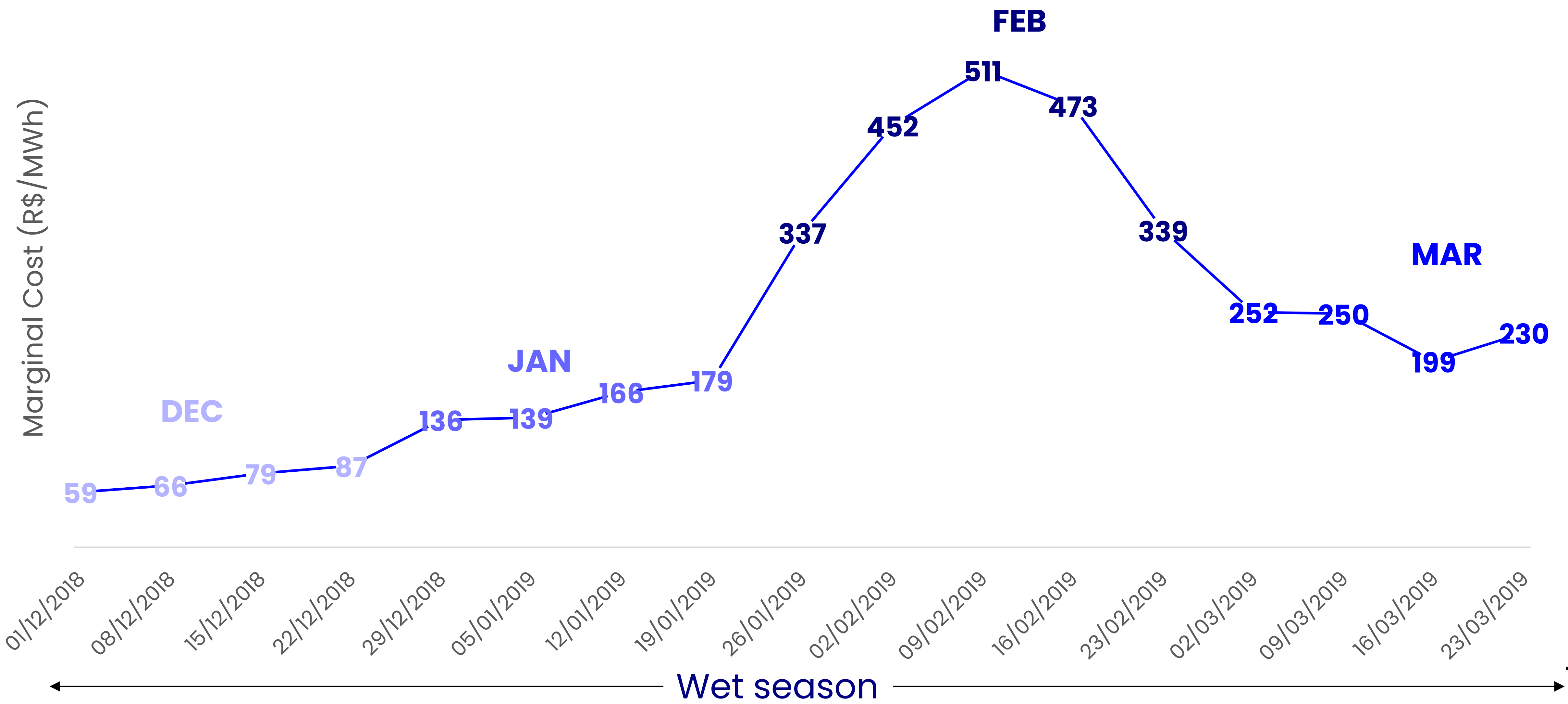
Someone is not happy with its results!

- Aggregated water values do not provide the signal for correct allocation of hydro resources
- Thermal plants dispatch needs to account for minimum up and down time
- Hydro plants can not operate in certain regions
- Reservoirs are depleting more than expected
- Marginal costs present volatility that does not make sense
- And so on...

Unreasonable volatility

Work in partnership with the Brazilian market operator (CCEE)

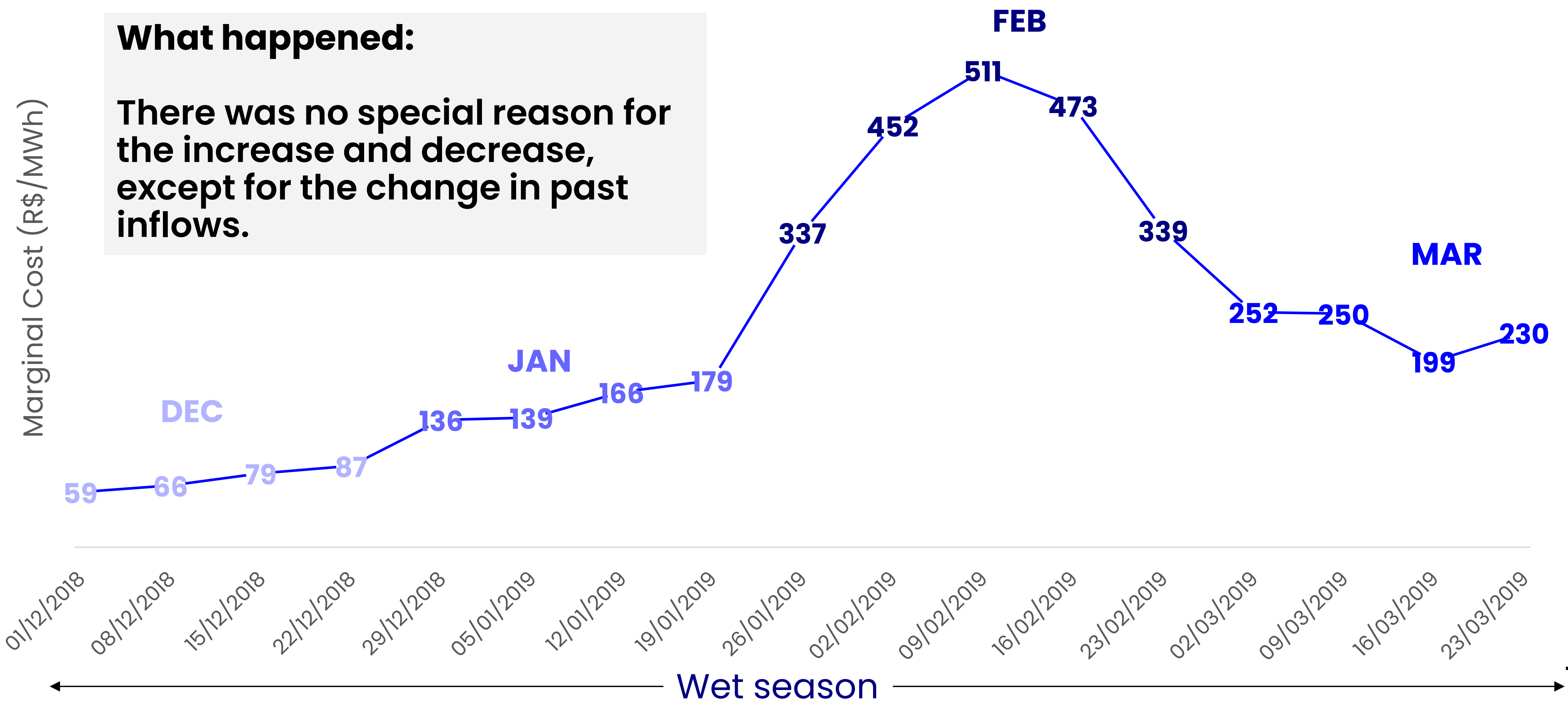
Example: Volatility of the System’s Marginal Cost



Example: Volatility of the System's Marginal Cost

What happened:

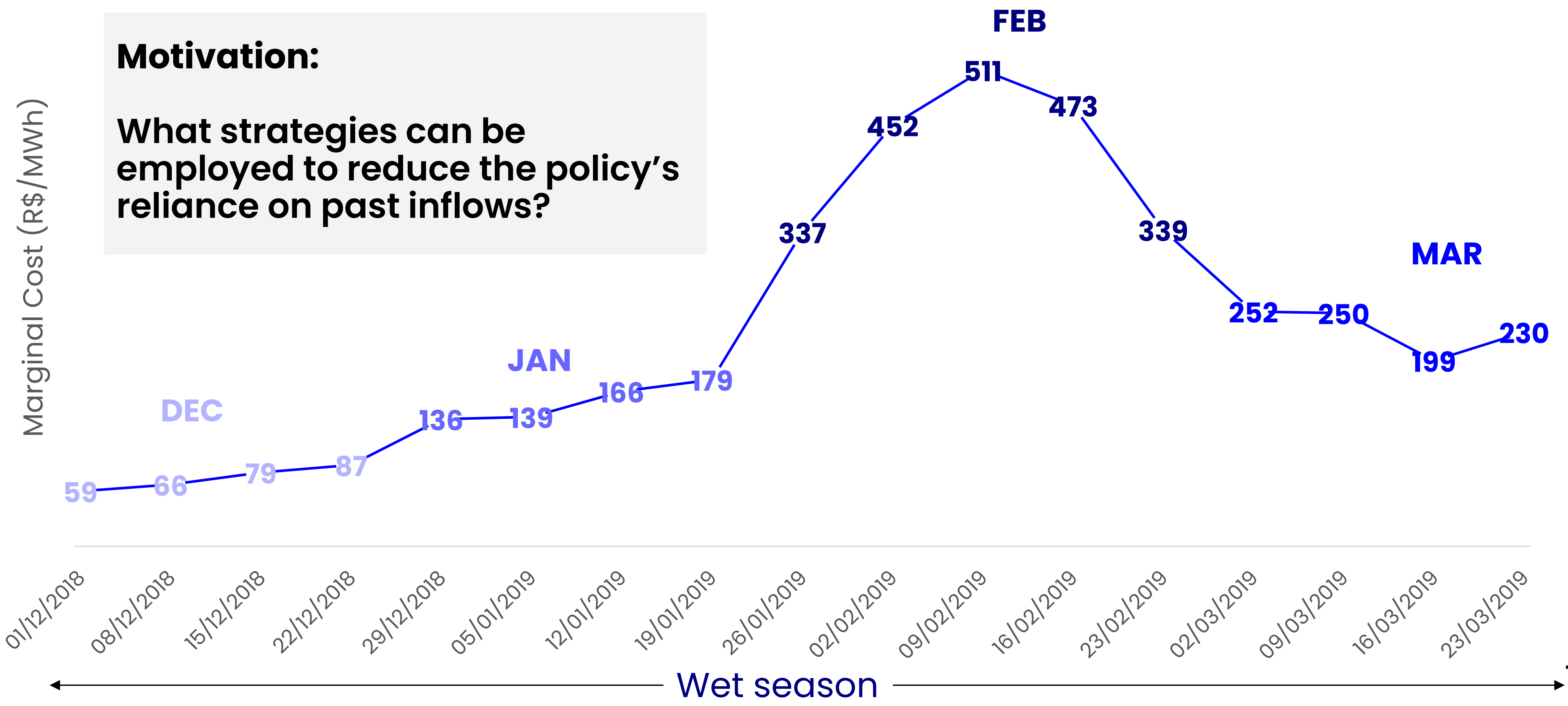
There was no special reason for the increase and decrease, except for the change in past inflows.



Example: Volatility of the System's Marginal Cost

Motivation:

What strategies can be employed to reduce the policy's reliance on past inflows?



What was happening that led to such sensitivity

SDDP builds policy iteratively

- Visit random scenarios in the Forward Pass
- Creates approximations of the future cost in the Backward Pass
- When considering temporal correlation for inflows, the previous inflows become a state
- This is a requirement to adjust the policy for any random scenario

PAR is a linear model with randomness on the residual

- It is considered one of the best models for generating inflow scenarios
- Its linearity with constant linear coefficients creates an accumulative over/under valuation

$$\dot{y}_t = \sum_{p=1}^{P_t} \emptyset_p^t\left(\dot{y}_{t-p}\right) + l_t$$

Why building a policy with PAR model may over/under value the inflow?

Assume a Hydro Power Plant with linear regression coefficient of 0.8 for all stages

$$\dot{y}_t = 0.8(\dot{y}_{t-1}) + l_t$$

If SDDP visits a scenario where the inflow in the first stage is 10, when creating a cut solving the second stage… there will be an inflow valuation based on an inflow equals to 10!

Say that in the second iteration the inflow is now 5, when evaluating the cut from the first iteration we are sure that there will be a reduction of 4 in all branches in the second stage.

Although we know that on some measure it is true that the expected inflow shall decrease, this is different than assuming that it changes for all branches in the same way.

Therefore, we were looking for an inflow model that accounts for uncertainty also in the linear coefficient.

Multidimensional Quantile Regression (MQR) Model

$$\dot{x}_t(\rho, \dot{x}_{t-1}) = M_t^\rho \dot{x}_{t-1} + D_t^\rho$$

Univariate version proposed by G. Pritchard in [2]

M_t^ρ and D_t^ρ depend on percentile ρ

[2]G. Pritchard, "Stochastic inflow modeling for hydropower scheduling problems," *Eur. J. Oper. Res.*, vol. 246, no. 2, pp. 496–504, Oct. 2015

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$\dot{x}_t(\rho, \dot{x}_{t-1}) = M_t^\rho \dot{x}_{t-1} + \boldsymbol{\Omega_t} D_t^\rho$

Load matrix $\boldsymbol{\Omega_t}$ to spatially couple generation inflow of several hydro plants

Obtained from the spatial correlation of the historical residuals $res_t = M_t^\rho x_{t-1} + D_t^\rho - x_t^{historical}$

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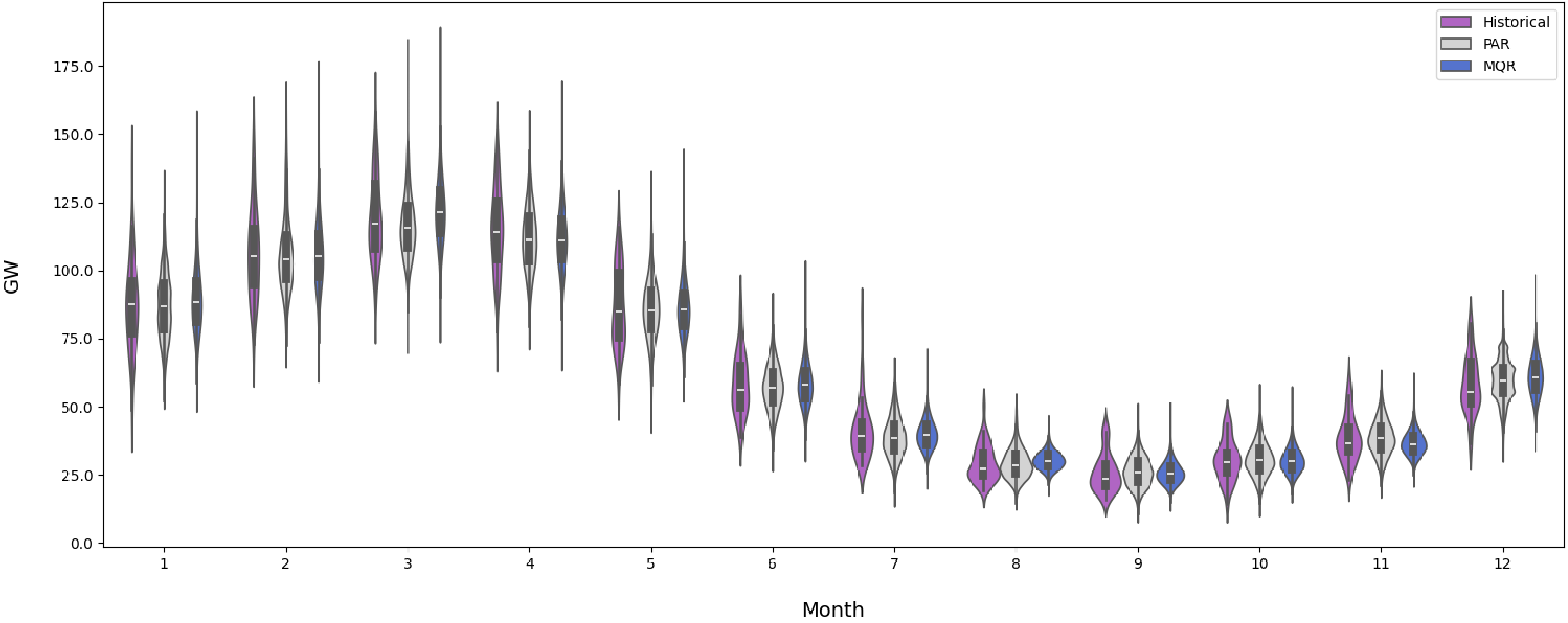
Important remarks:

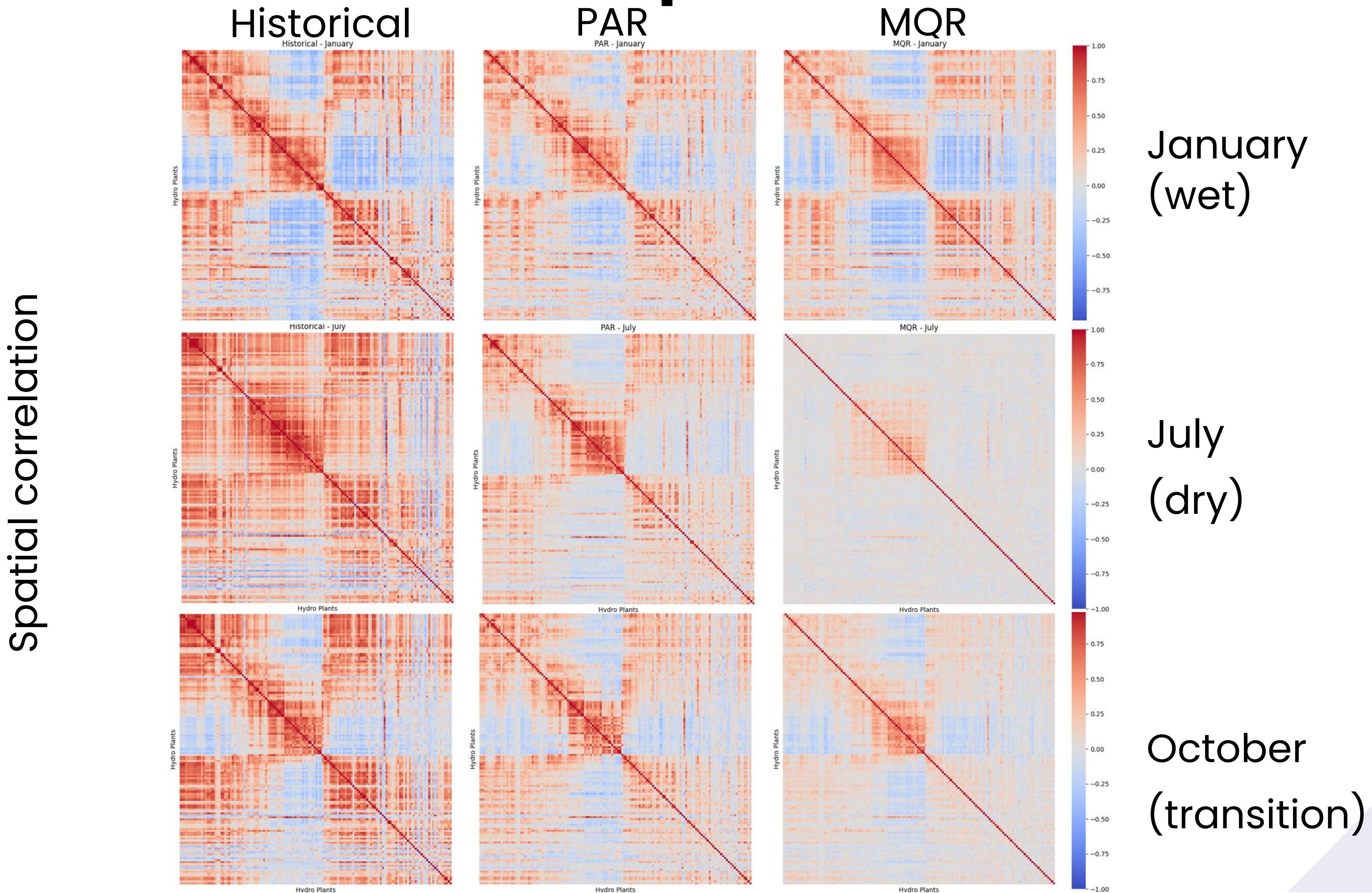
First-order model
Random parameters in both linear and constant coefficients
Transformation to guarantee strict positive inflow values!

[2]G. Pritchard, “Stochastic inflow modeling for hydropower scheduling problems,” *Eur. J. Oper. Res.*, vol. 246, no. 2, pp. 496–504, Oct. 2015

Statistical Moments Assessment



Spatial Correlation Comparison



MQR model characteristics

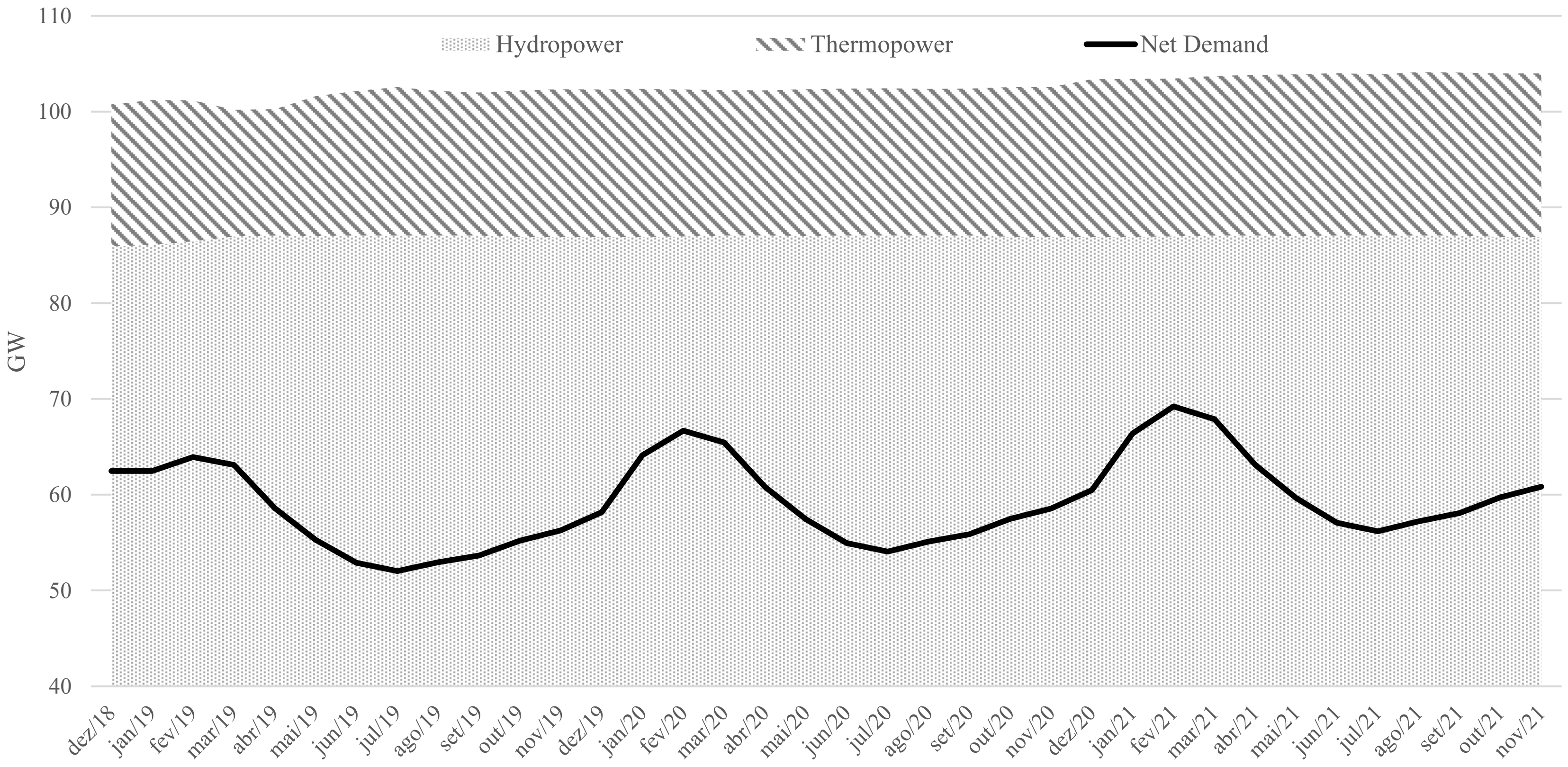
- Multiple percentiles provides a comprehensive representation of the inflow sample space → **Suitable** for policy construction during the **Backward pass**

- Weaker temporal correlation w.r.t the PAR model → **Not as effective** for obtaining trial points during the **Forward pass**

Numerical experiments

Simplified and large-scale instance of the Brazilian LTHS problem

- 130 Thermal plants – 16 GWa
- 140 Hydro plants (79 run-of-river) – 87 GWa
- Net demand – 59 GWa



Optimization Settings

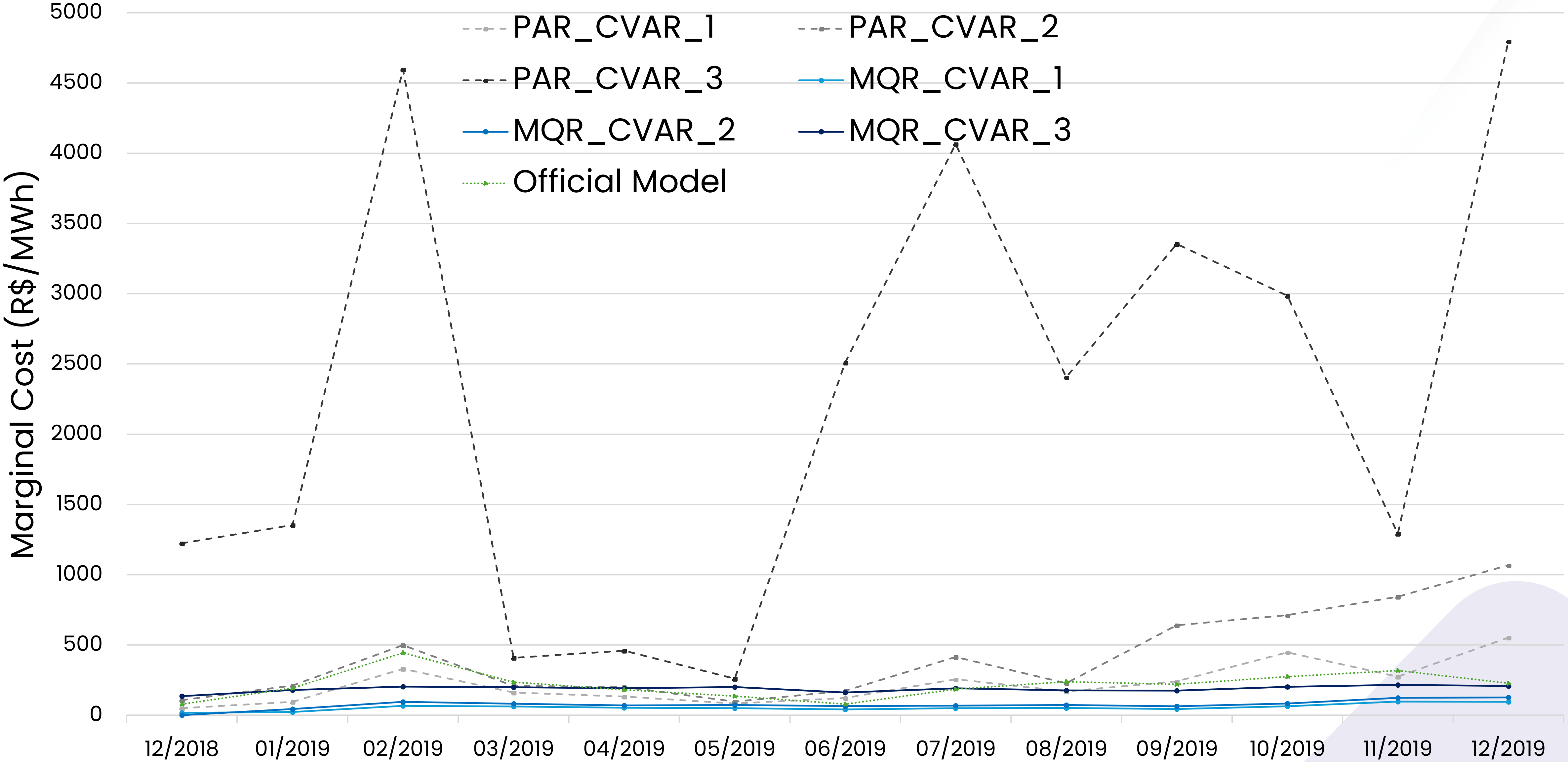
- **Forward pass:**
  - PAR model
- **Backward pass**:
  - PAR model /MQR model
- **Conditional Value-at-Risk ($CVaR_{1-\alpha}$)**
  - CVAR_1: α = 25% and λ = 35% (Standard)
  - CVAR_2: α = 15% and λ = 40% (+)
  - CVAR_3: α = 20% and λ = 90% (+++)
- **Computational Setup:**
  - 20 parallel forward passes per iteration / 20 scenarios per stage
  - 200 iterations, resulting in 4,000 cuts
  - Cut selection strategy
  - Horizon: 36 months divided into monthly time-steps

Simulation Framework

- **13-month rolling-horizon simulation**
  - 5,000 out-of-sample PAR-generated inflow scenarios
  - Historical observed inflows

Policy	Forward Inflow model	Backward Inflow model	CVAR (α, λ)
PAR_CVaR_1	PAR	PAR	(25,35)
PAR_CVaR_2	PAR	PAR	(15,40)
PAR_CVaR_3	PAR	PAR	(20,90)
MQR_CVaR_1	PAR	MQR	(25,35)
MQR_CVaR_2	PAR	MQR	(15,40)
MQR_CVaR_3	PAR	MQR	(20,90)

Rolling-horizon simulation under observed inflows



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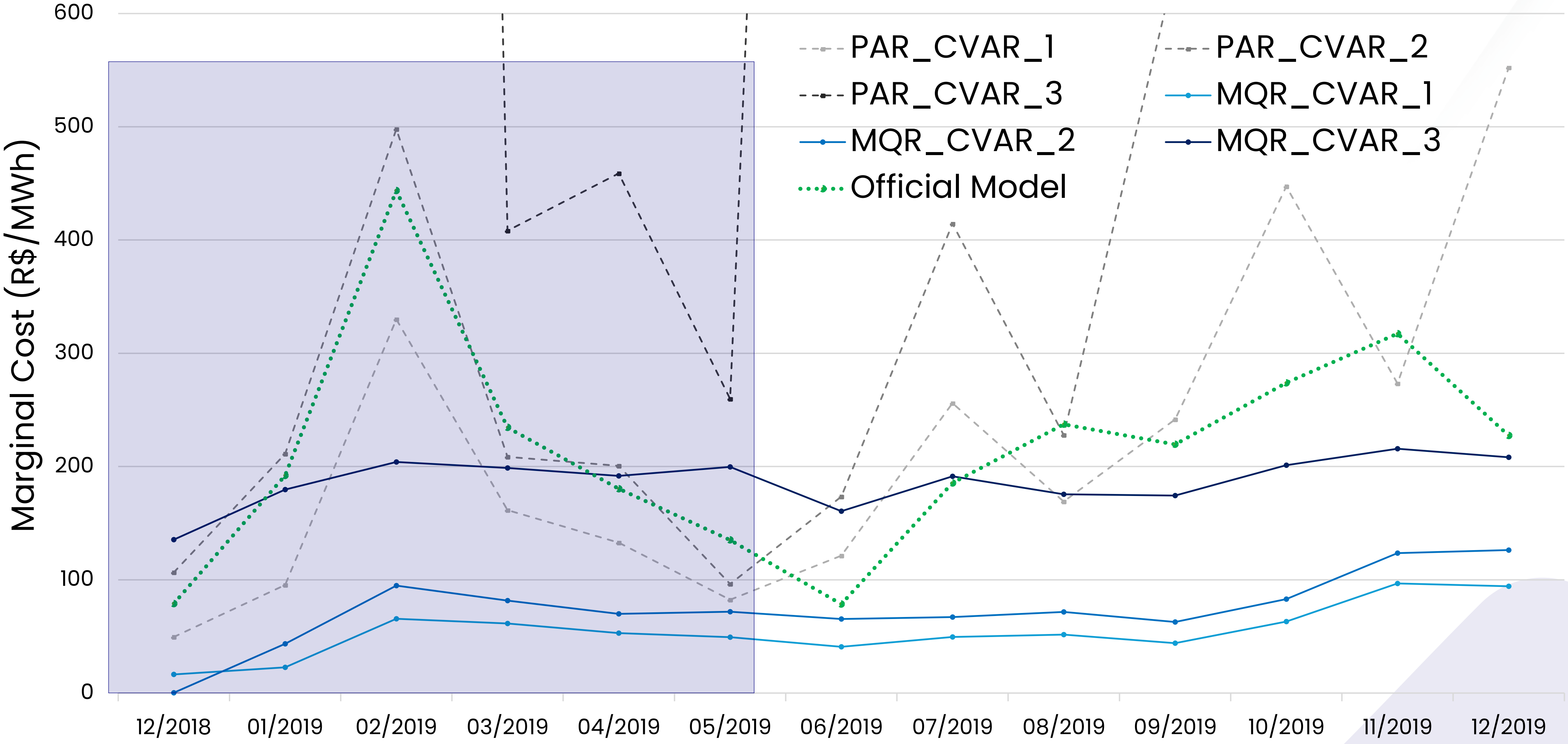
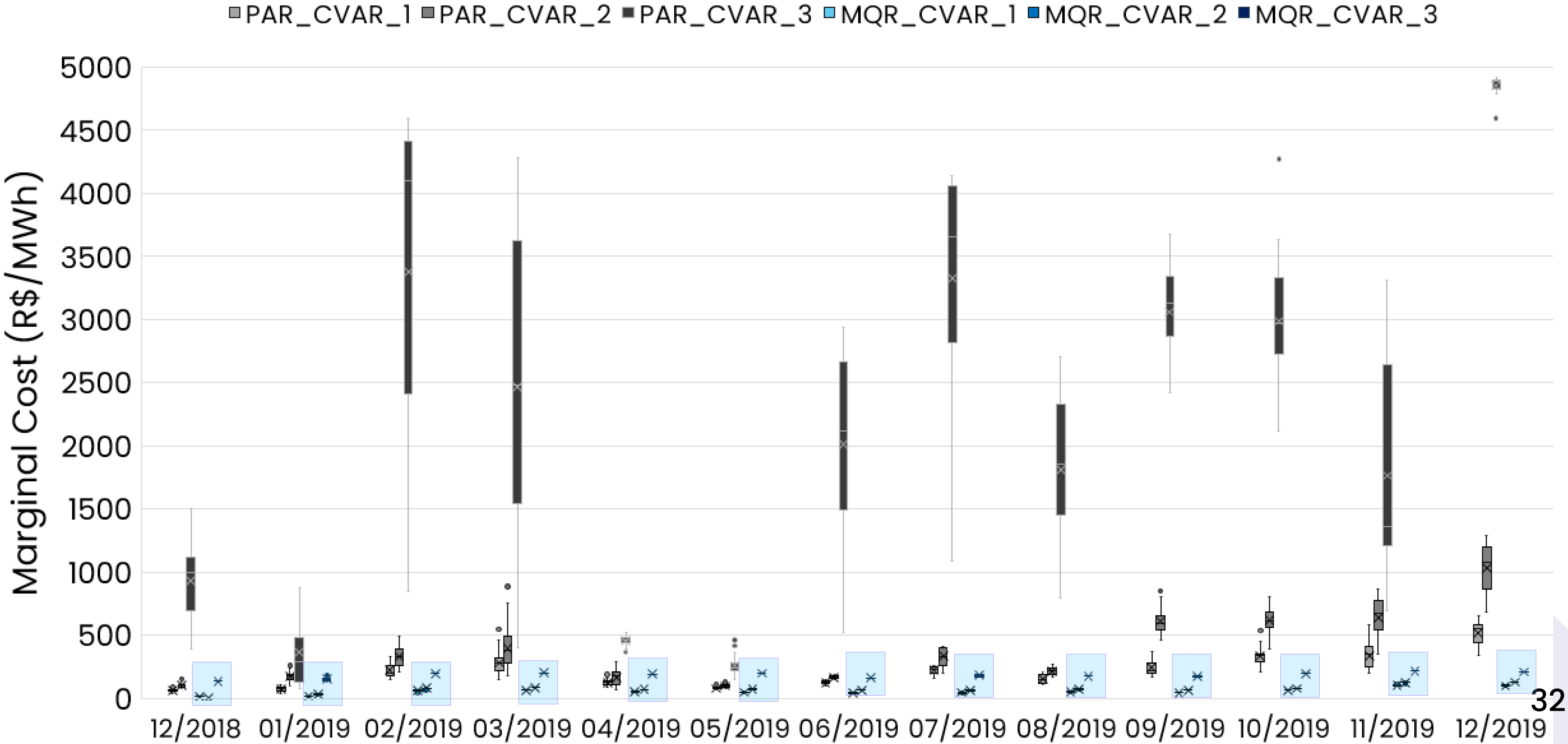


Rolling-horizon simulation under out-of-sample inflows



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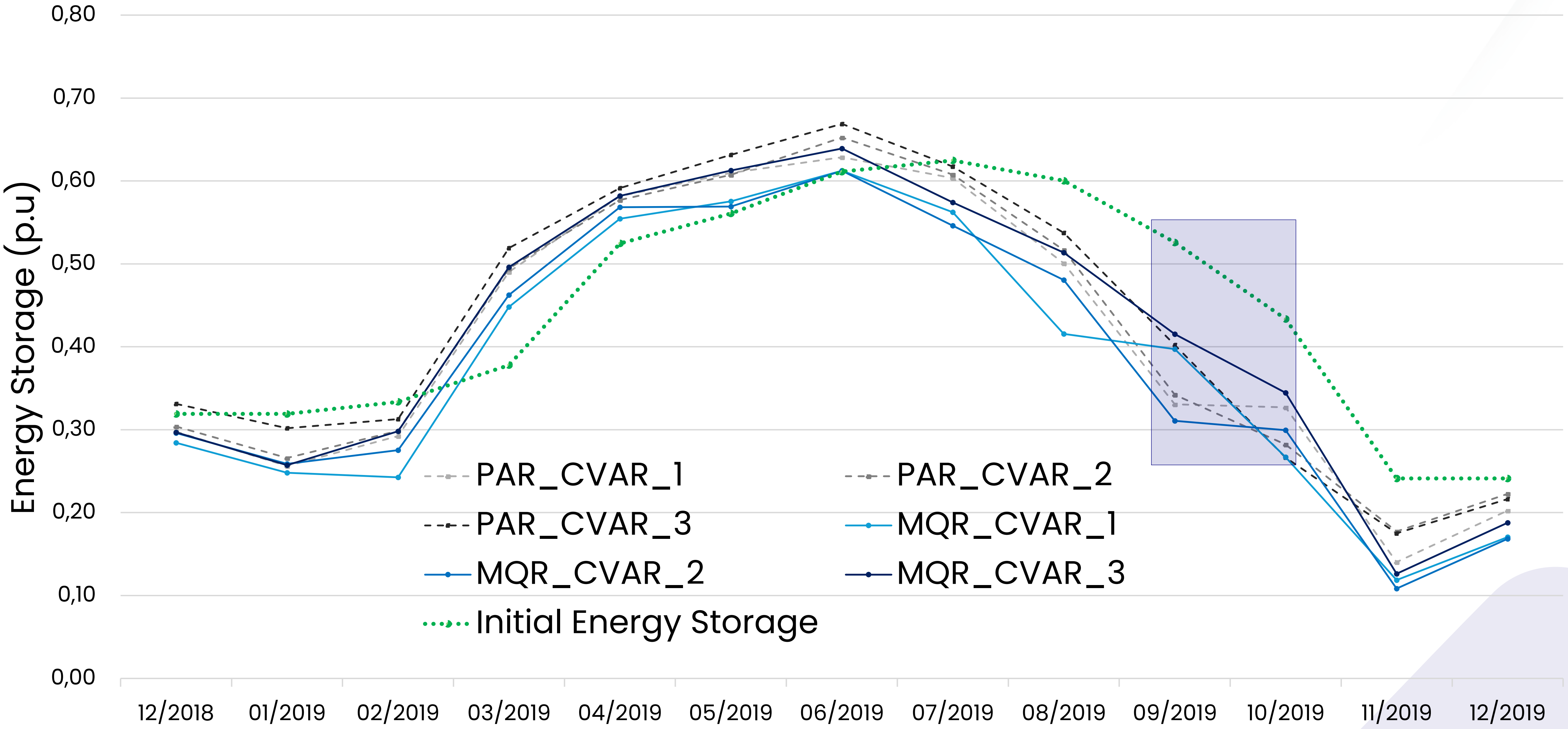

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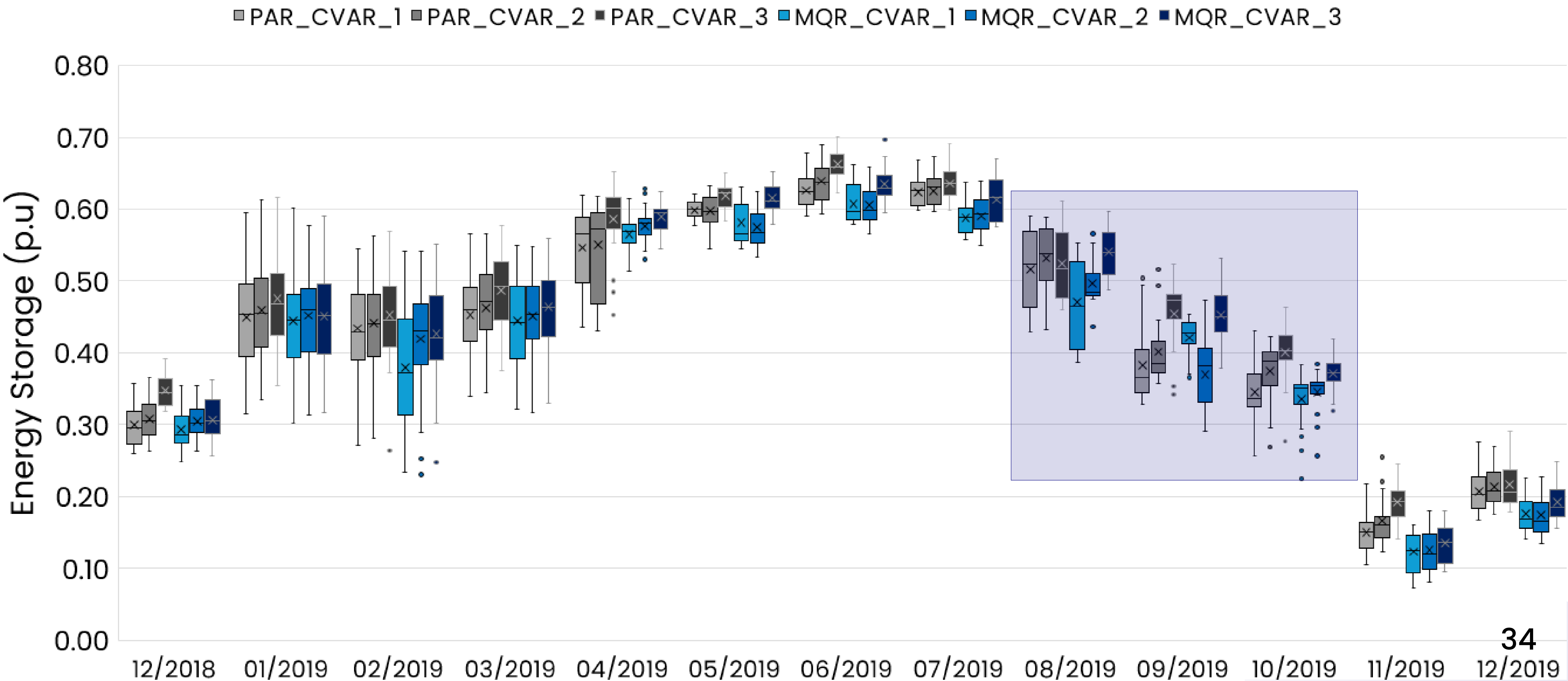


Conclusions

- **MQR-based policies compared to the PAR-based policies:**
  - Maintains similar reservoir levels when paired with an adequate risk-aversion calibration
  - Mitigate the volatility observed in energy prices and thermal generation
  - Less sensitive to CVaR parameter settings
- **Challenges:**
  - Complexity: Two inflow models for a SDDP execution (two formulations!)
  - Adequate risk-aversion calibration

Soon a paper will be available with details!

Norus | Questions?

Thank you!

vitor.matos@norus.com.br

What is to come…

Toward a Unified SDDP Framework for Heterogeneous Stochastic Dynamics

- **Ignoring uncertainties from other sources than Inflows, may lead to regrettable decisions and increase the price volatility**
- **We are developing in Power Fusion a generalized setup that allow for various stochastic process modelling to be used in the SDDP algorithm**

Benefits:

- Decision process allows multiple independent stochastic process
- Modelling uncertainty in different timeframes in the same problem
- Physical and Statistical model can be combined
- Allow independent decision on time steps and stages

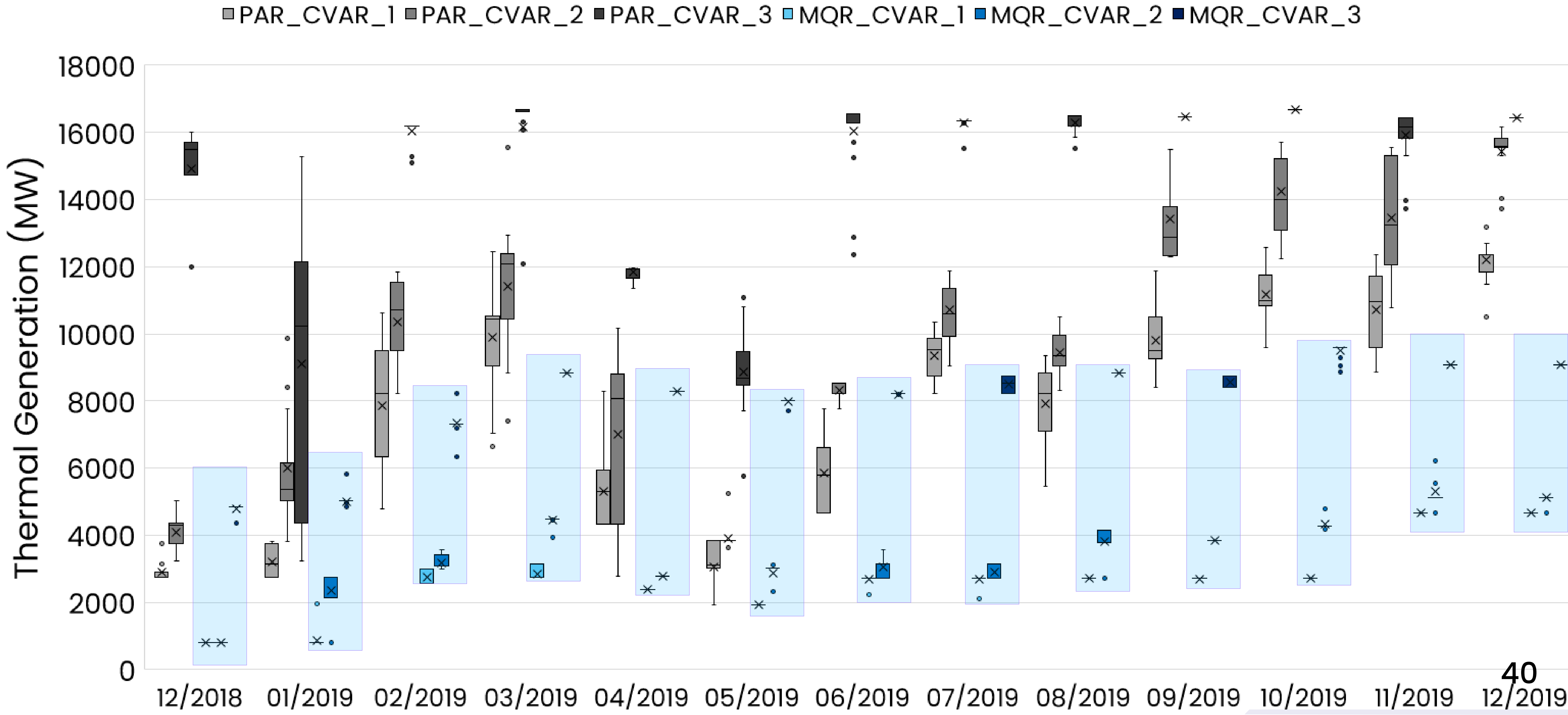
Challenges:

- Growth of the combinatorial problem with an increased number of random variables with different stochastic processes
- Need to use a simplified multi-cut to allow cut sharing

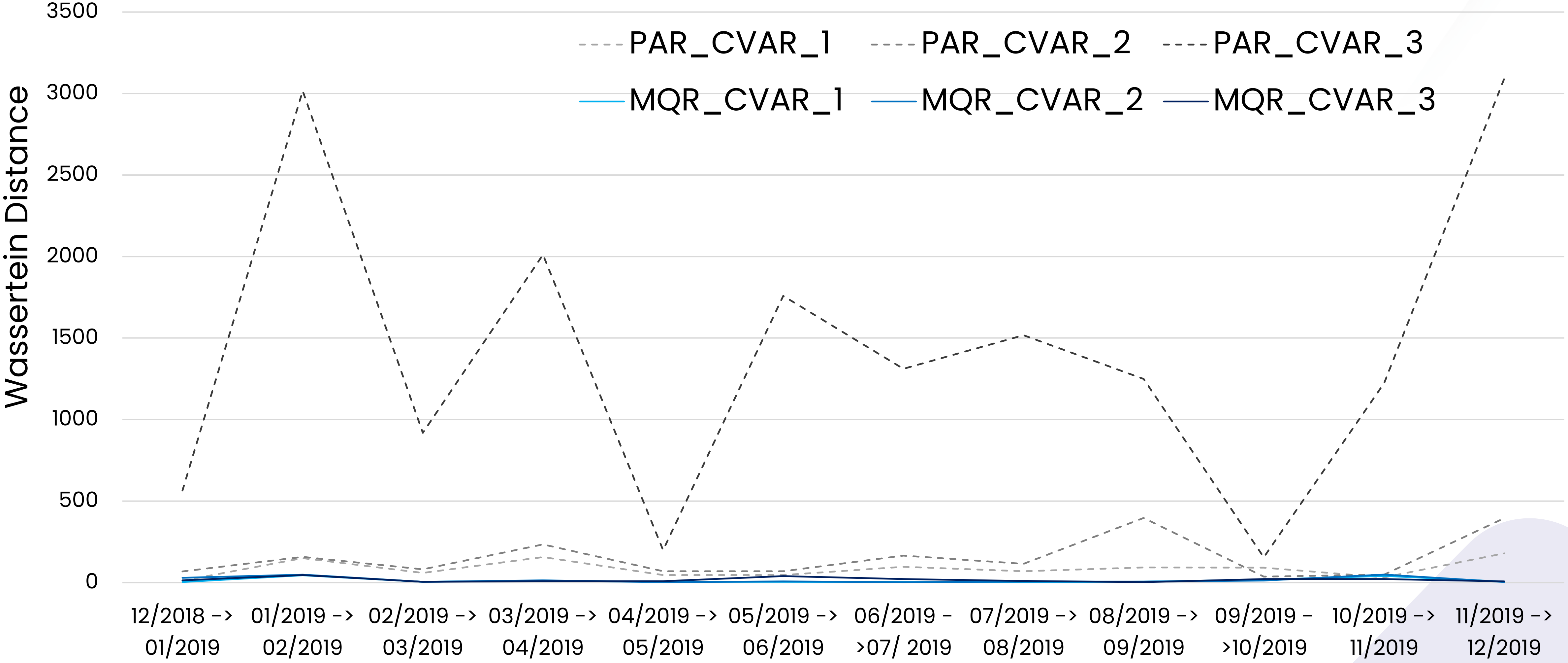
Unreasonable volatility

Work in partnership with the Brazilian market operator (CCEE)

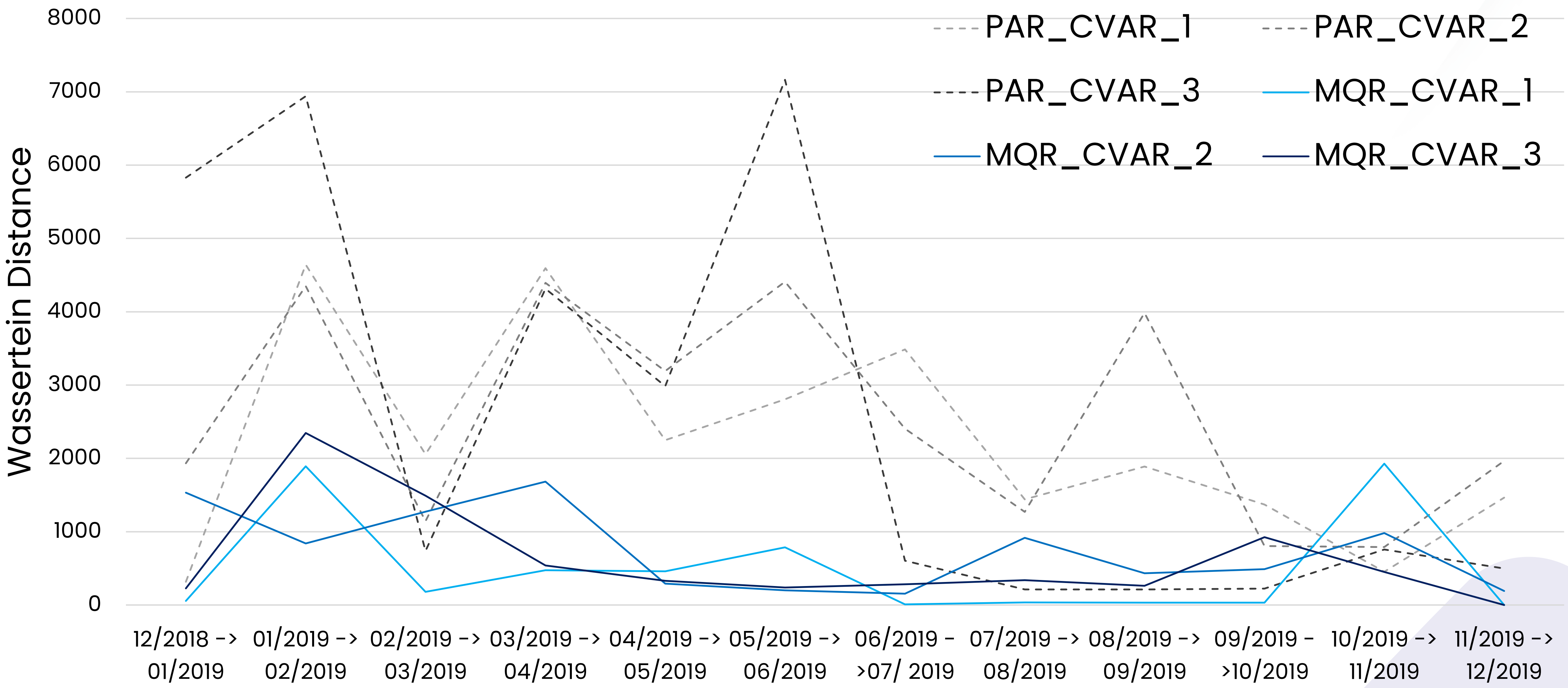
Rolling-horizon simulation under out-of-sample inflows



Volatility – Marginal Cost



Volatility – Thermal Generation



Volatility – Storage Energy